Estimation

What is an estimate?

An estimate is a measure taken from a sample that approximates a **population parameter** μ, p, σ^2, σ . Population parameters are also known as true values from a population. If we were to conduct a census of population size N, we would be able to determine the exact values for the following.

Population Mean

$$\mu = \frac{\sum x}{N}$$

Population Proportion

$$p = \frac{x}{N}$$

Population Variance

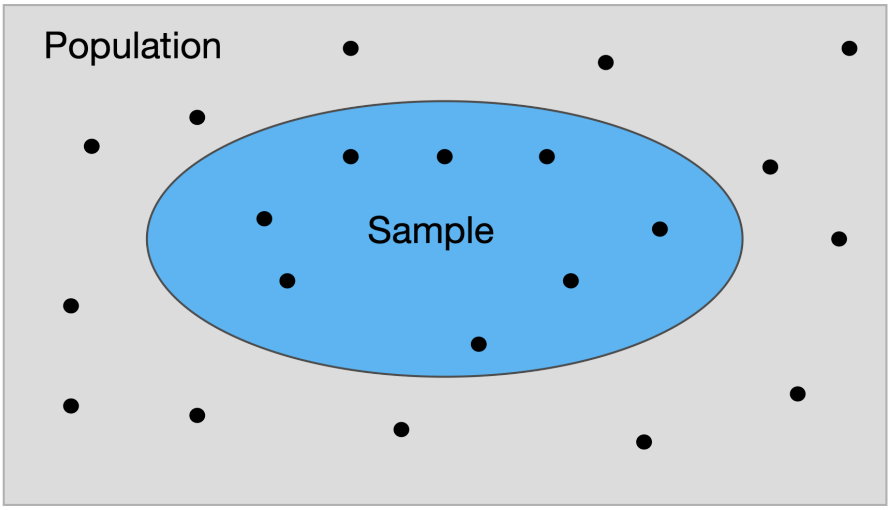
$$\sigma^2 = \frac{N\sum x^2 - (\sum x)^2}{N(N-1)}$$

Population Standard Deviation

$$\sigma = \sqrt{\frac{N\sum x^2 - (\sum x)^2}{N(N-1)}}$$

Why do we need to conduct an estimate when we can always take a census?

We can't always take a census. In fact we rarely conduct a census as it is too time consuming and too expensive. This is why we need to approximate population parameter with **point estimates** $\bar{x}, \bar{p}, s^2, s$. These point estimates are also known as **statistics** and they come from a sample of sample size n.



Sample Mean

$$\bar{x} = \frac{\sum x}{n}$$

Sample Proportion

$$\bar{p} = \frac{x}{n}$$

Sample Variance

$$s^2 = \frac{n\sum x^2 - (\sum x)^2}{n(n-1)}$$

Sample Standard Deviation

$$s = \sqrt{\frac{n\sum x^2 - (\sum x)^2}{n(n-1)}}$$

Confidence Intervals aka Interval Estimates

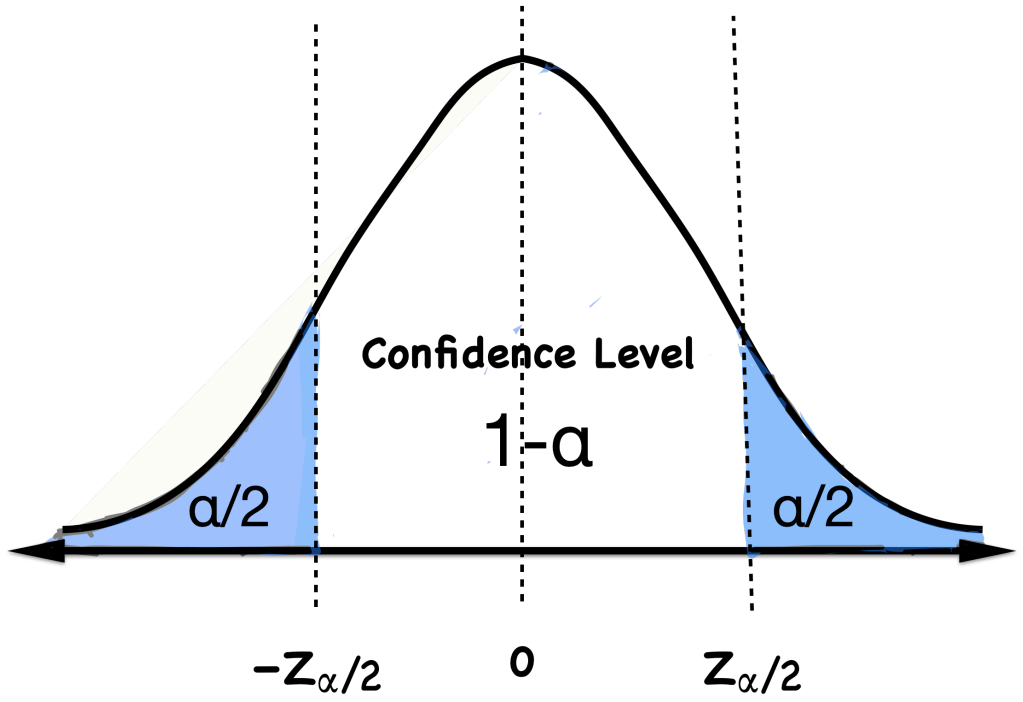
An interval estimate can also be determined to provide a range of values in which a **statistic** can approximate a **population (true) parameter**.

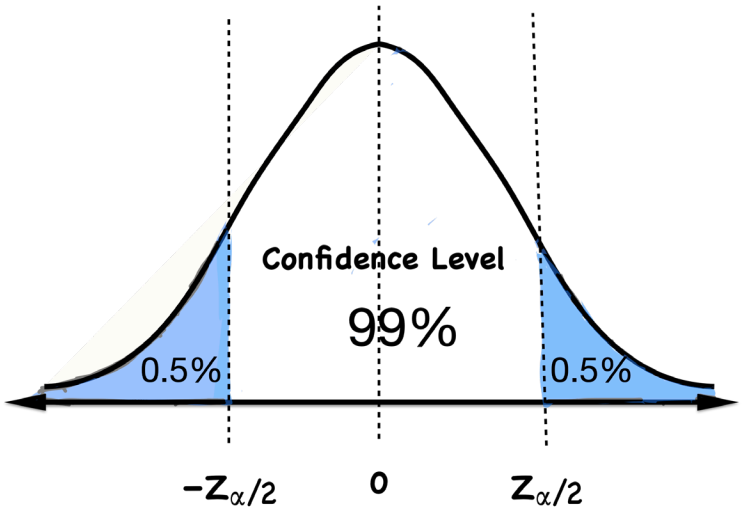
Estimate a True Proportion

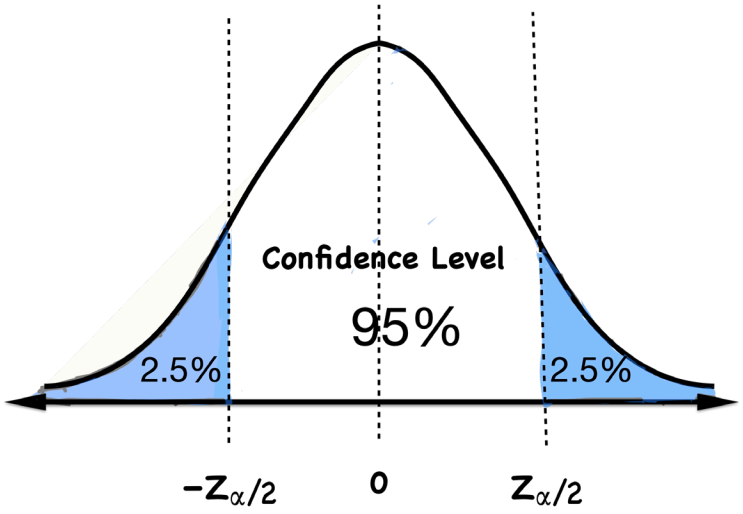
$$\bar{p} - E < p < \bar{p} + E$$

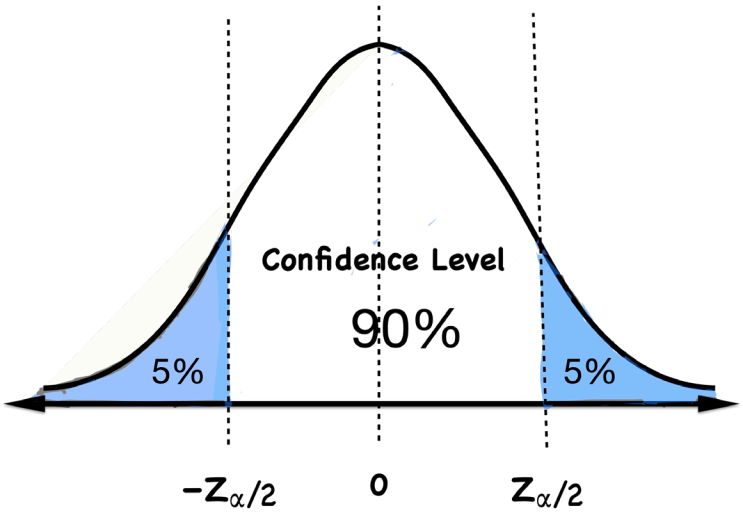
$$E = z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

The z Distribution




Confidence Level
99%
0.5%
0.5%
$-Z_{\alpha/2}$
0
$Z_{\alpha/2}$


Confidence Level
95%
2.5%
2.5%
$-Z_{\alpha/2}$
0
$Z_{\alpha/2}$


Confidence Level
90%
5%
5%
$-Z_{\alpha/2}$
0
$Z_{\alpha/2}$

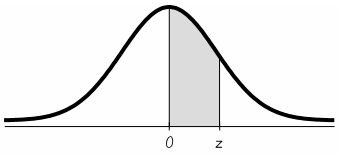


TABLE A-2 Standard Normal (z) Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495 *	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949 *	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.10 and higher	.4999									

NOTE: For values of z above 3.09, use 0.4999 for the area.

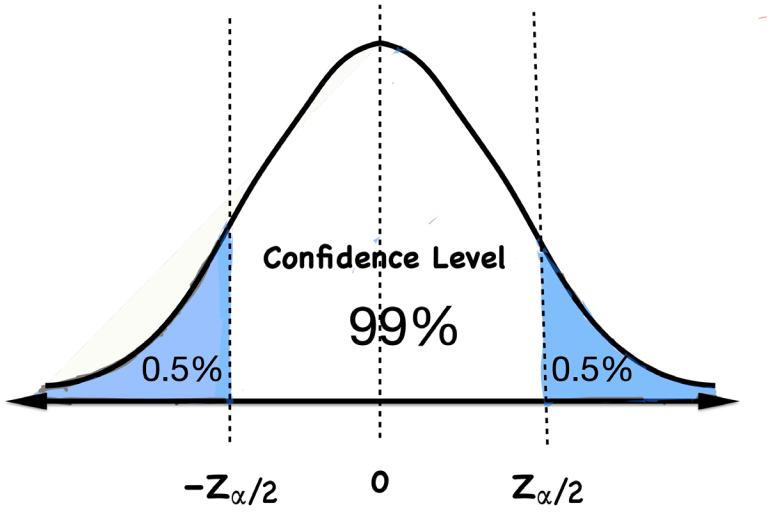
*Use these common values that result from interpolation:

z score	Area
1.645	0.4500
2.575	0.4950

From Frederick C. Mosteller and Robert E. K. Rourke, *Sturdy Statistics,* 1973, Addison-Wesley Publishing Co., Reading, MA. Reprinted with permission of Frederick Mosteller.

Fast Food Order Accuracy

In a study of fast food order accuracy, Mickey D's had 28 orders that were not accurate out of 402 orders. Use the 99% confidence level to estimate the true proportion of orders that are not accurate.

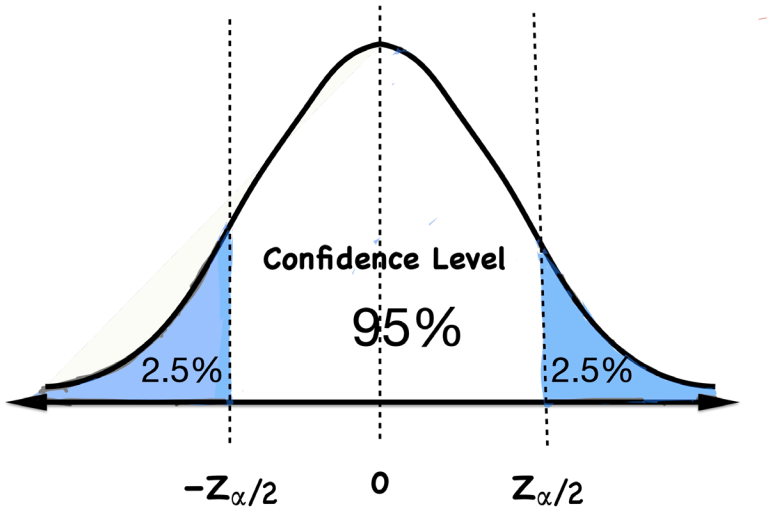


TI-83 or TI-84 Plus Confidence Interval

1. Press **STAT** then select **TESTS** in the top menu.
2. Select **1-PropZint** in the menu and press **ENTER.**
3. Enter the number of successes x, number of observations n, and the Confidence Level (C-level).
4. Select **Calculate** and press **ENTER.**

Medical Malpractice Lawsuits

In a study of 1300 randomly selected medically malpractice lawsuits, it was found that 835 were dropped or dismissed. Construct a 95% confidence interval estimating the true proportion of lawsuits that are dropped or dismissed.



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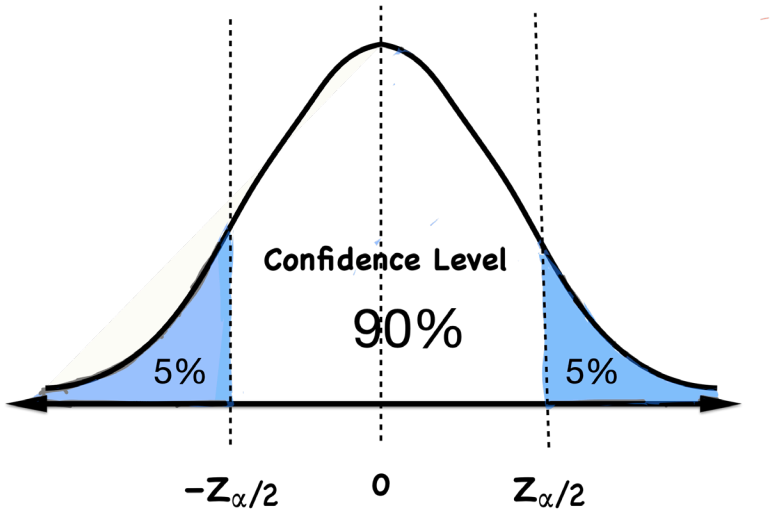
2. Select **1-PropZint** in the menu and press **ENTER.**
3. Enter the number of successes x, number of observations n, and the Confidence Level (C-level).
4. Select **Calculate** and press **ENTER.**

Smoking

In a program designed to help people to stop smoking, 200 patients were given sustained care in with 85.5% were no longer smoking and were biochemically confirmed. Use the 90% confidence level to estimate the true proportion of people who receive sustained care who will stop smoking.



TI-83 or TI-84 Plus Confidence Interval

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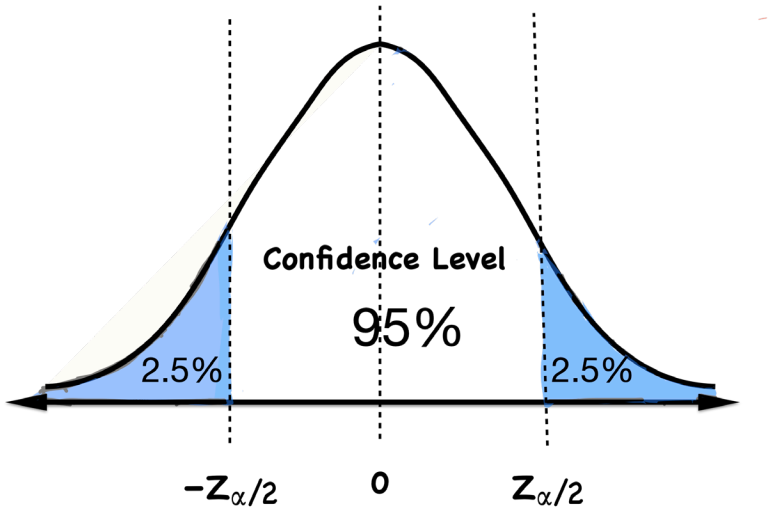
3. Enter the number of successes x, number of observations n, and the Confidence Level (C-level).
4. Select **Calculate** and press **ENTER.**

Astrology

A Sociologist gives a survey to 500 people on whether they believe in Astrology and finds that 38.2% indicate that they do. Use the 95% confidence level to estimate the true proportion of people who believe in Astrology.



TI-83 or TI-84 Plus Confidence Interval

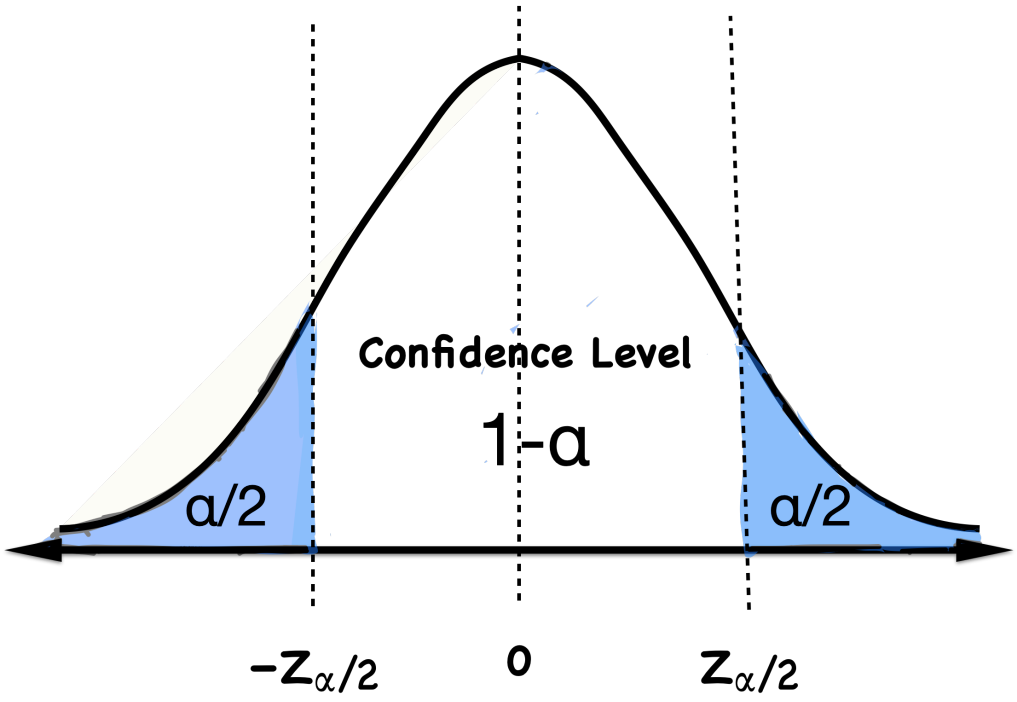
1. Press **STAT** then select **TESTS** in the top menu.
2. Select **1-PropZint** in the menu and press **ENTER.**
3. Enter the number of successes x, number of observations n, and the Confidence Level (C-level).
4. Select **Calculate** and press **ENTER.**

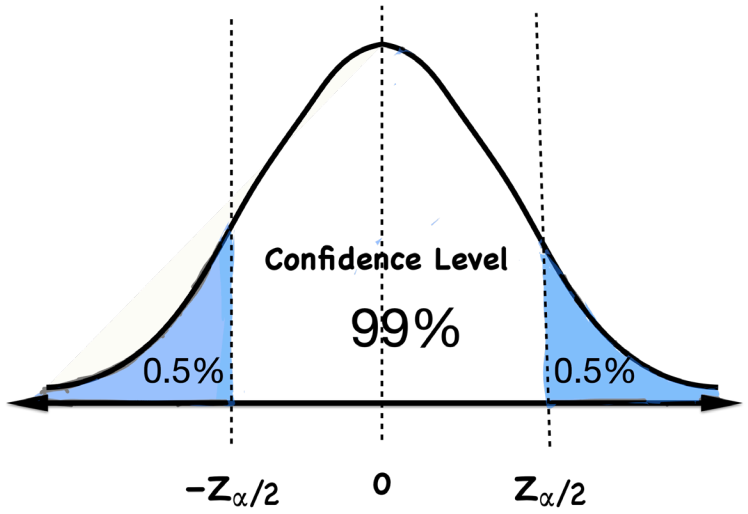
Estimate a True Mean

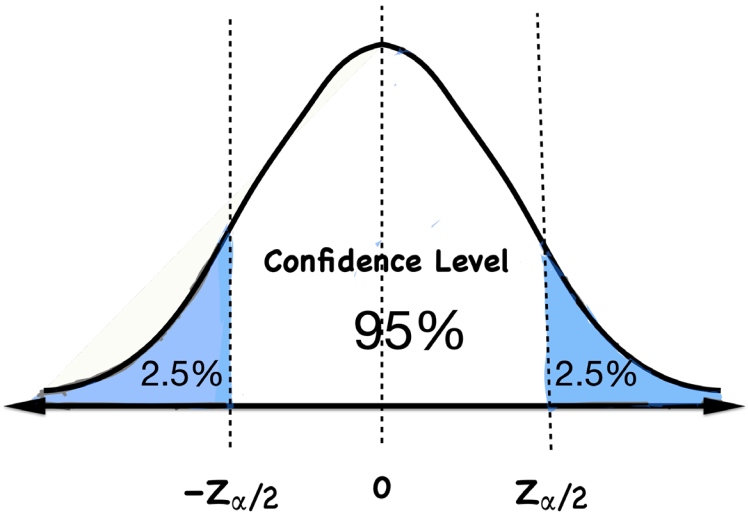
$$\bar{x} - E < \mu < \bar{x} + E$$

$E = z\alpha_{/2}\frac{\sigma}{\sqrt{n}}$ σ known or $n > 30$ large Sample

The z Distribution




Confidence Level
99%
0.5%
0.5%
$-Z_{\alpha/2}$
0
$Z_{\alpha/2}$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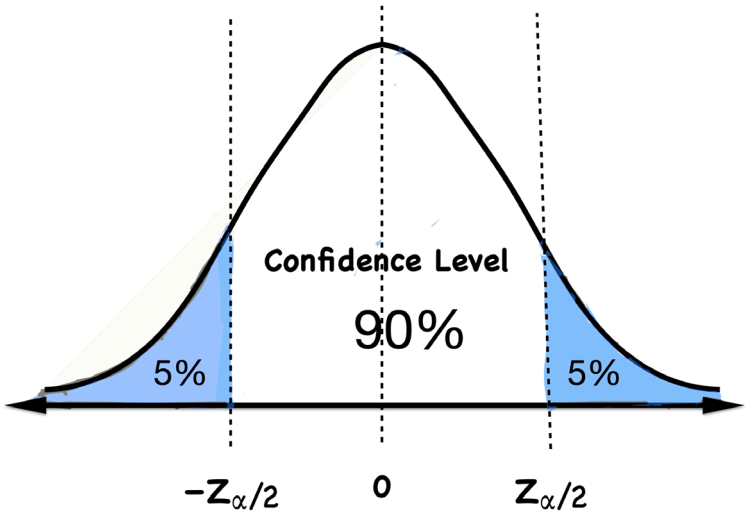
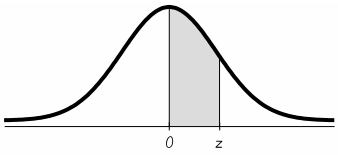
$Z_{\alpha/2}$


Confidence Level
90%
5%
5%
$-Z_{\alpha/2}$
0
$Z_{\alpha/2}$



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0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
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0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
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2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949 *	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.10 and higher	.4999									

NOTE: For values of z above 3.09, use 0.4999 for the area.

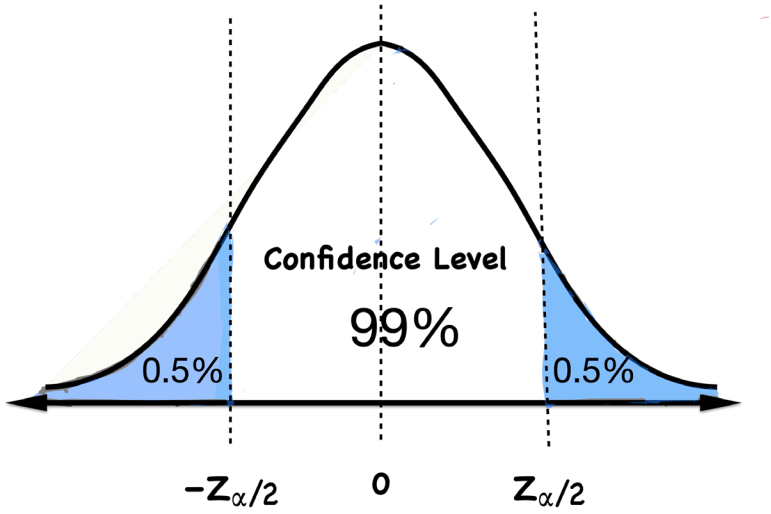
*Use these common values that result from interpolation:

z score	Area
1.645	0.4500
2.575	0.4950

From Frederick C. Mosteller and Robert E. K. Rourke, *Sturdy Statistics,* 1973, Addison-Wesley Publishing Co., Reading, MA. Reprinted with permission of Frederick Mosteller.

Years In College

A sample of 600 College students indicated it took them a mean of 6.5 years with a standard deviation of 1.8 years to earn a bachelor's degree. Use the 99% confidence level to estimate the true mean years it takes to earn a bachelor's degree.

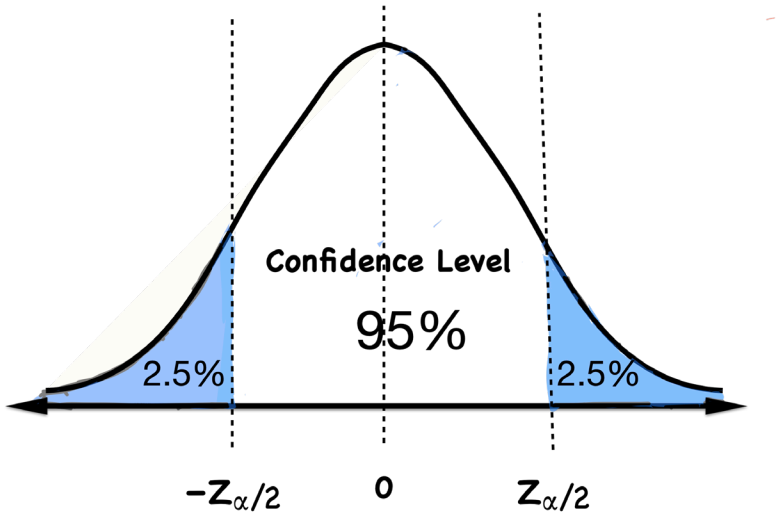


TI-83 or TI-84 Plus Confidence Interval

1. Press **STAT** then select **TESTS** in the top menu.
2. Select **Zinterval** in the menu and select **STATS.**
3. Enter the the standard deviation ($\sigma \; or \; s$), the sample mean $\bar{x}$, sample size n, Confidence Level (C-Level).
4. Select **Calculate** and press **ENTER.**

Celebrity Net Worth

A sample of 65 celebrity's were surveyed and reported a mean net worth of 200 Million Dollars with a standard deviation of 28.5 million dollars. Use the 95% confidence level to estimate the true mean net worth of a celebrity.

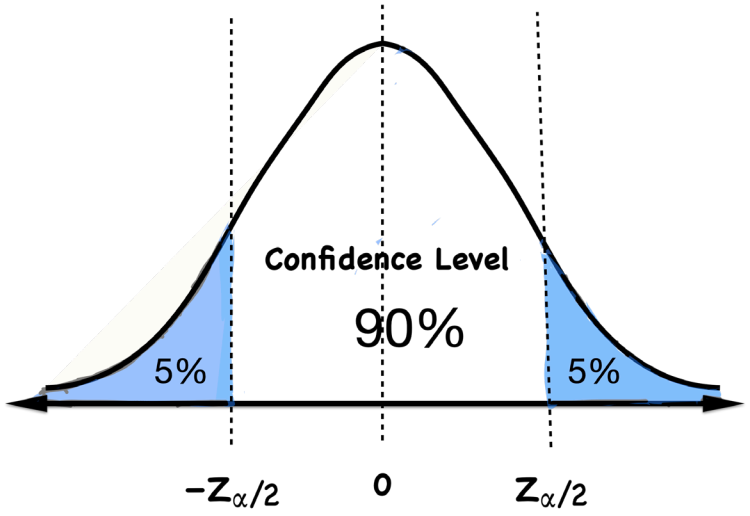


TI-83 or TI-84 Plus Confidence Interval

1. Press **STAT** then select **TESTS** in the top menu.
2. Select **Zinterval** in the menu and select **STATS.**
3. Enter the the standard deviation ($\sigma \; or \; s$), the sample mean $\bar{x}$, sample size n, Confidence Level (C-Level).
4. Select **Calculate** and press **ENTER.**

IQ of Attorney's

A survey of 120 Attorney's report a mean IQ score of 108.6 with a standard deviation of 12.5. Use the 90% confidence level to estimate the true mean IQ score for an attorney.



TI-83 or TI-84 Plus Confidence Interval

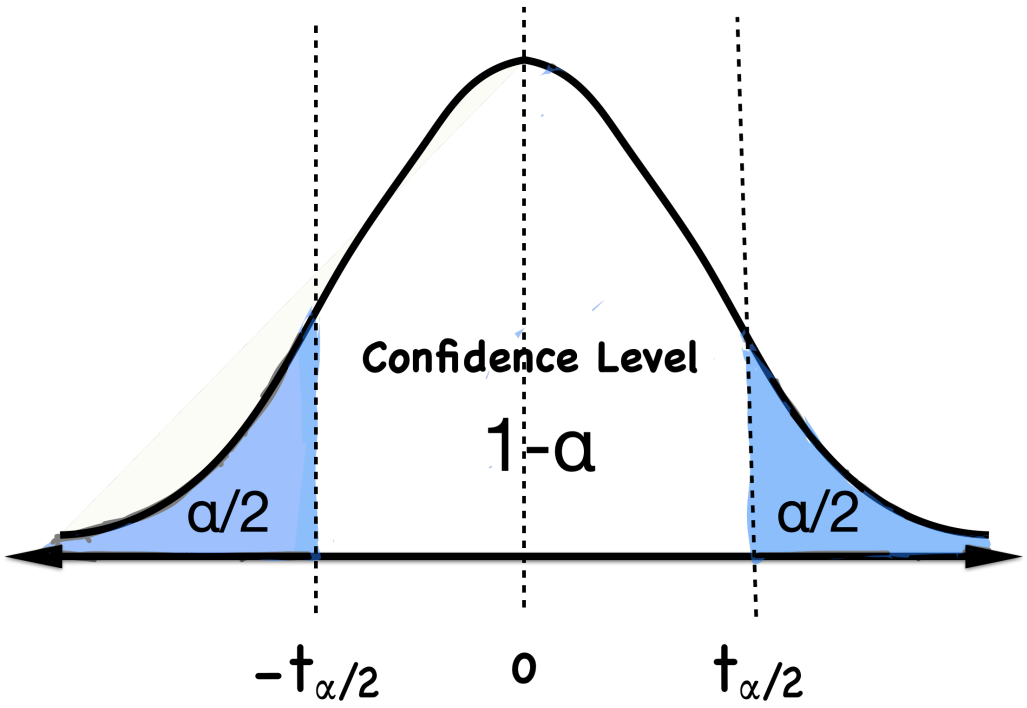
1. Press **STAT** then select **TESTS** in the top menu.
2. Select **Zinterval** in the menu and select **STATS.**
3. Enter the the standard deviation ($\sigma \; or \; s$), the sample mean $\bar{x}$, sample size n, Confidence Level (C-Level).
4. Select **Calculate** and press **ENTER.**

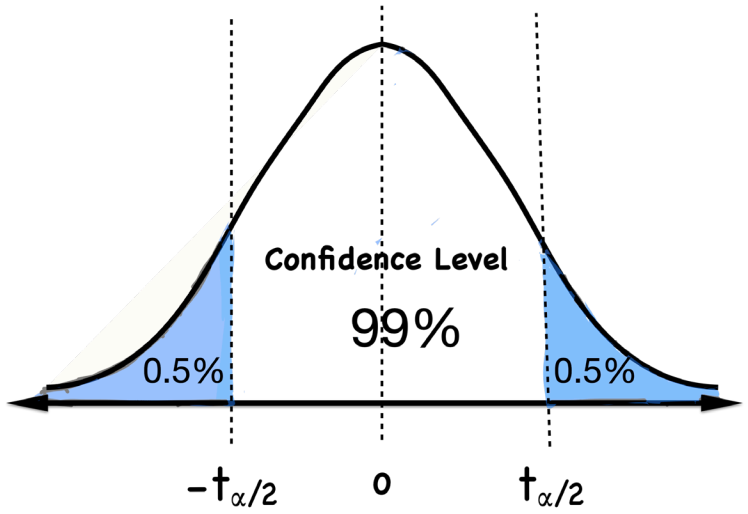
Estimate a True Mean for a small sample $n \leq 30$

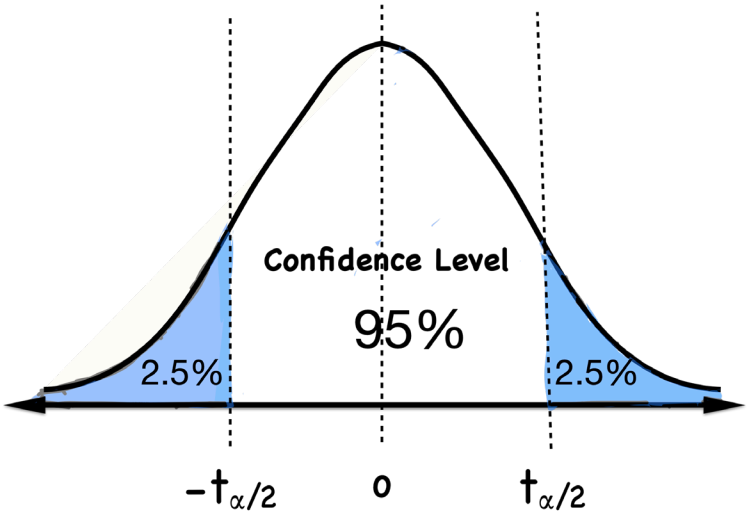
$$\bar{x} - E < \mu < \bar{x} + E$$

$E = t\alpha/_2 \frac{s}{\sqrt{n}}$ σ unknown or $n \leq 30$

The t Distribution aka Student's Distribution




Confidence Level
99%
0.5%
0.5%
$-t_{\alpha/2}$
0
$t_{\alpha/2}$


Confidence Level
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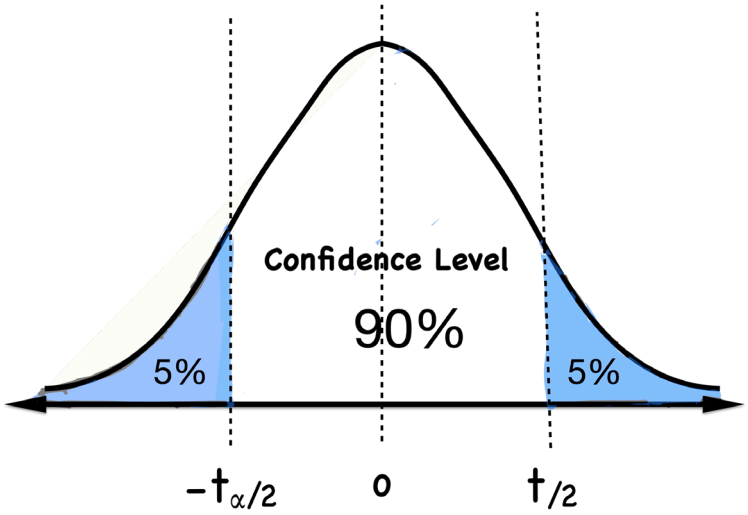
Confidence Level
90%
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$-t_{\alpha/2}$
0
$t_{/2}$

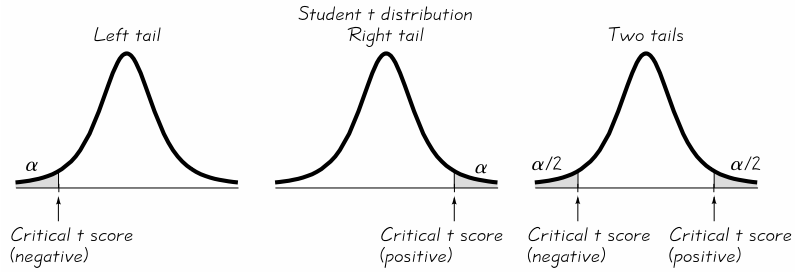
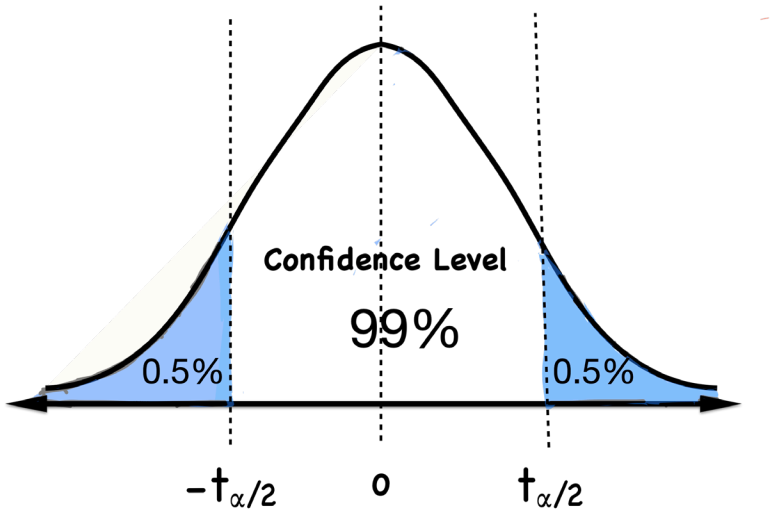


TABLE A-3 *t* Distribution

	α					
Degrees of Freedom	.005 (one tail) 0.1 (two tails)	.01 (one tail) .02 (two tails)	.025 (one tail) .05 (two tails)	.05 (one tail) .10 (two tails)	.10 (one tail) .20 (two tails)	.25 (one tail) .50 (two tails)
1	63.657	31.821	12.706	6.314	3.078	1.000
2	9.925	6.965	4.303	2.920	1.886	.816
3	5.841	4.541	3.182	2.353	1.638	.765
4	4.604	3.747	2.776	2.132	1.533	.741
5	4.032	3.365	2.571	2.015	1.476	.727
6	3.707	3.143	2.447	1.943	1.440	.718
7	3.500	2.998	2.365	1.895	1.415	.711
8	3.355	2.896	2.306	1.860	1.397	.706
9	3.250	2.821	2.262	1.833	1.383	.703
10	3.169	2.764	2.228	1.812	1.372	.700
11	3.106	2.718	2.201	1.796	1.363	.697
12	3.054	2.681	2.179	1.782	1.356	.696
13	3.012	2.650	2.160	1.771	1.350	.694
14	2.977	2.625	2.145	1.761	1.345	.692
15	2.947	2.602	2.132	1.753	1.341	.691
16	2.921	2.584	2.120	1.746	1.337	.690
17	2.898	2.567	2.110	1.740	1.333	.689
18	2.878	2.552	2.101	1.734	1.330	.688
19	2.861	2.540	2.093	1.729	1.328	.688
20	2.845	2.528	2.086	1.725	1.325	.687
21	2.831	2.518	2.080	1.721	1.323	.686
22	2.819	2.508	2.074	1.717	1.321	.686
23	2.807	2.500	2.069	1.714	1.320	.685
24	2.797	2.492	2.064	1.711	1.318	.685
25	2.787	2.485	2.060	1.708	1.316	.684
26	2.779	2.479	2.056	1.706	1.315	.684
27	2.771	2.473	2.052	1.703	1.314	.684
28	2.763	2.467	2.048	1.701	1.313	.683
29	2.756	2.462	2.045	1.699	1.311	.683
Large (z)	2.575	2.326	1.960	1.645	1.282	.675

Years In College

A sample of 25 College students indicated it took them a mean of 6.8 years with a standard deviation of 0.9 years to earn a bachelor's degree. Use the 99% confidence level to estimate the true mean years it takes to earn a bachelor's degree.

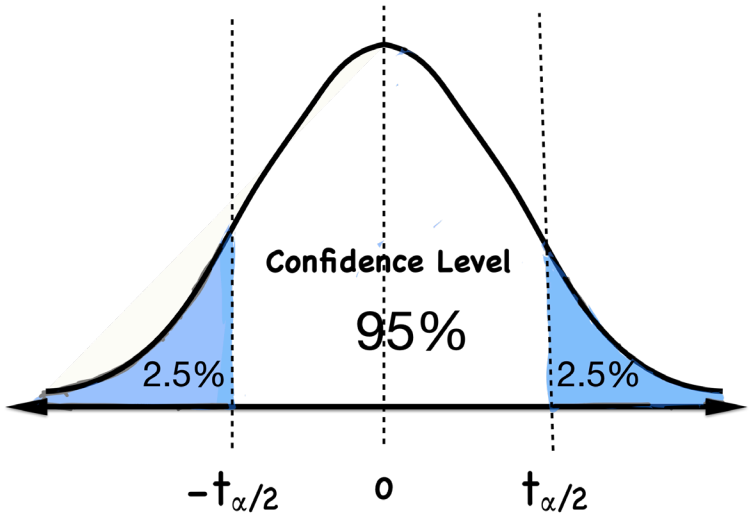


TI-83 or TI-84 Plus Confidence Interval

1. Press **STAT** then select **TESTS** in the top menu.
2. Select **Tinterval** in the menu and select **STATS.**
3. Enter the the standard deviation ($\sigma \; or \; s$), the sample mean $\bar{x}$, sample size n, Confidence Level (C-Level).
4. Select **Calculate** and press **ENTER.**

Insomnia Treatment

A clinical trial was conducted to test the effectiveness of a drug on insomnia patients that had a mean wake time of 108.6 minutes without the drug. A sample of 18 insomnia patients who took the drug reported a mean wake time of 96.2 minutes with a standard deviation of 43.6 minutes. Use the 95% confidence level to estimate the true mean wake time.



TI-83 or TI-84 Plus Confidence Interval

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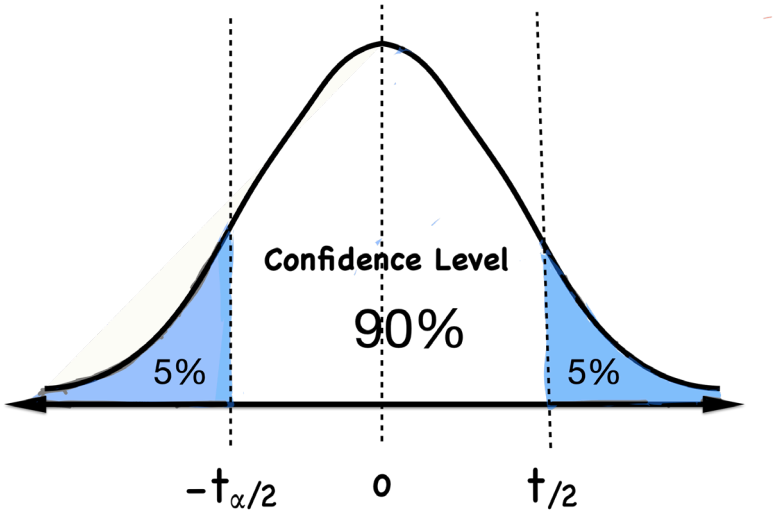
2. Select **Tinterval** in the menu and select **STATS.**
3. Enter the the standard deviation ($\sigma \; or \; s$), the sample mean $\bar{x}$, sample size n, Confidence Level (C-Level).
4. Select **Calculate** and press **ENTER.**

Age of Male College Students

A survey of 25 male college students report a mean age of 26.6 years with a standard deviation of 3.8 years. Use the 90% confidence level to estimate the true mean age of male college students.



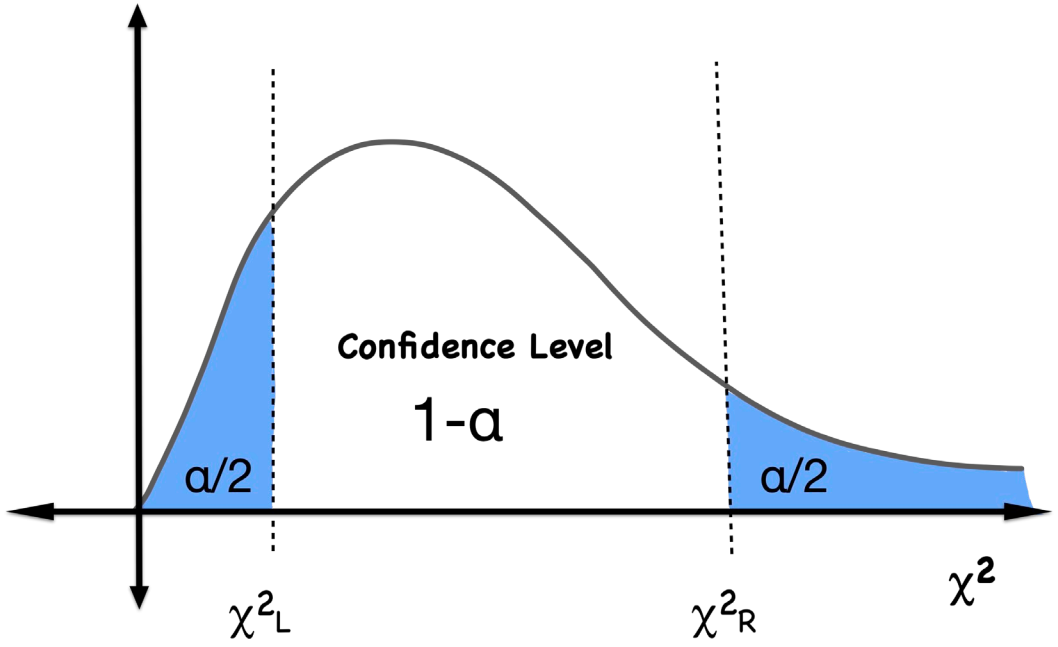
TI-83 or TI-84 Plus Confidence Interval

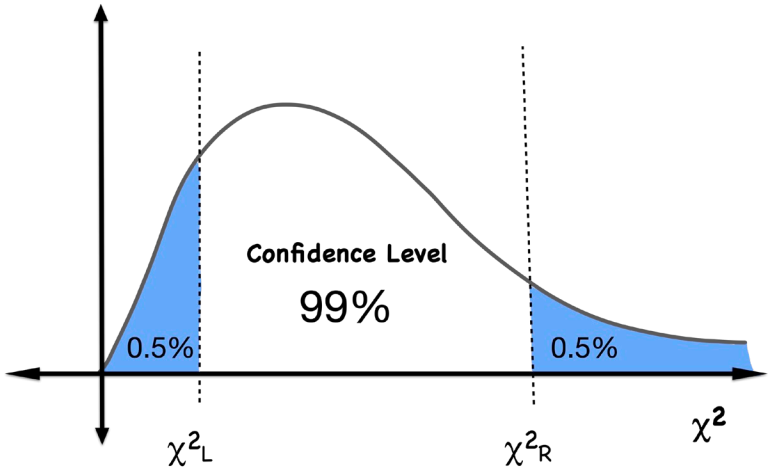
1. Press **STAT** then select **TESTS** in the top menu.
2. Select **Tinterval** in the menu and select **STATS.**
3. Enter the the standard deviation ($\sigma \; or \; s$), the sample mean $\bar{x}$, sample size n, Confidence Level (C-Level).
4. Select **Calculate** and press **ENTER.**

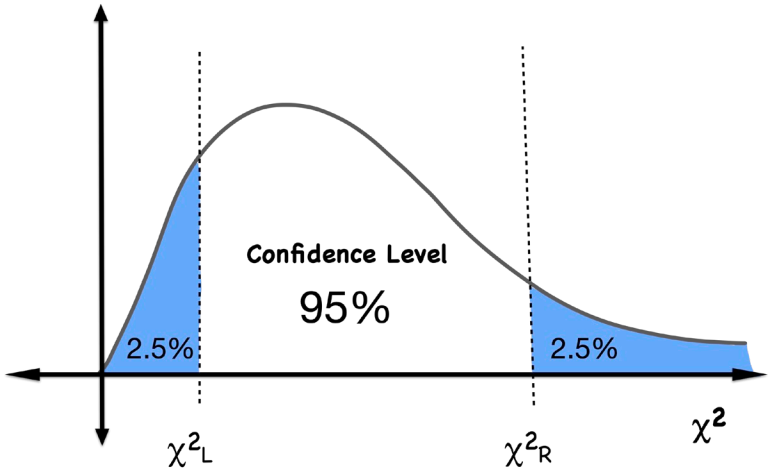
Estimate a True Variance

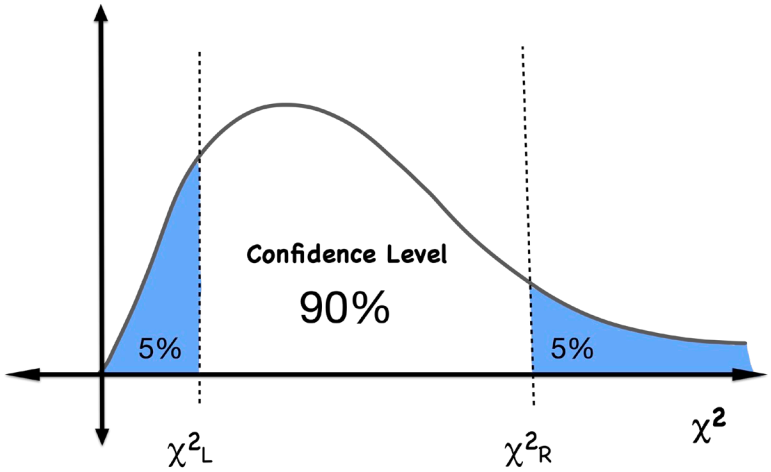
$$\frac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2}$$

The Chi Square Distribution χ^2




Confidence Level
99%
0.5%
0.5%
χ^2_L
χ^2_R
χ^2


Confidence Level
95%
2.5%
2.5%
χ^2_L
χ^2_R
χ^2


Confidence Level
90%
5%
5%
χ^2_L
χ^2_R
χ^2

Formulas and Tables

for *Elementary Statistics, Eighth Edition*, by Mario F. Triola

TABLE A-4 Chi-Square (χ^2) Distribution

Degrees of Freedom	Area to the Right of the Critical Value									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

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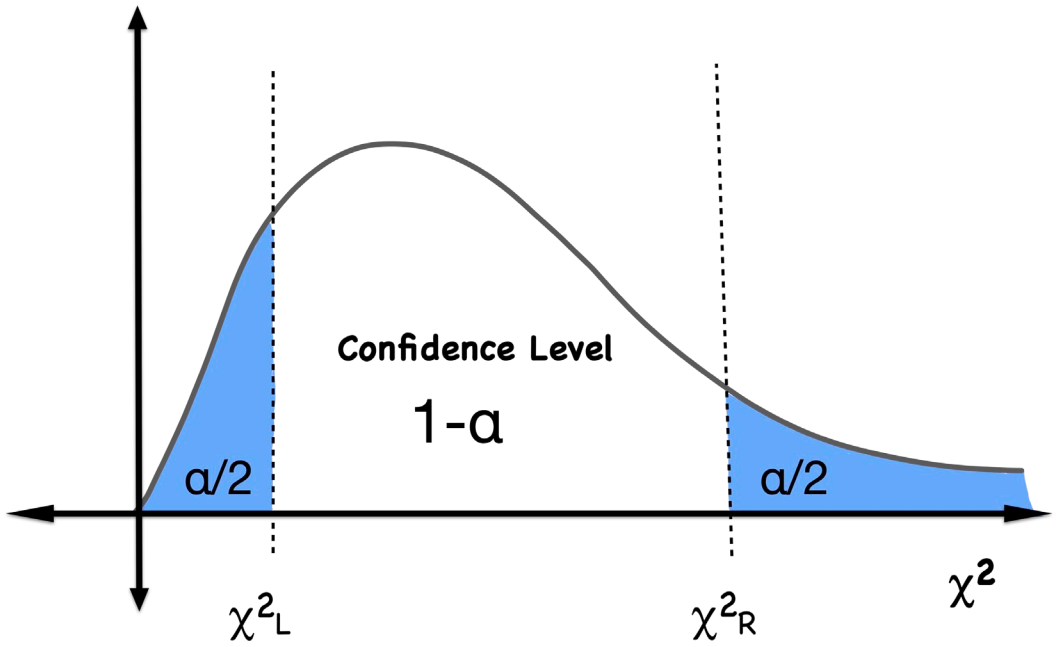
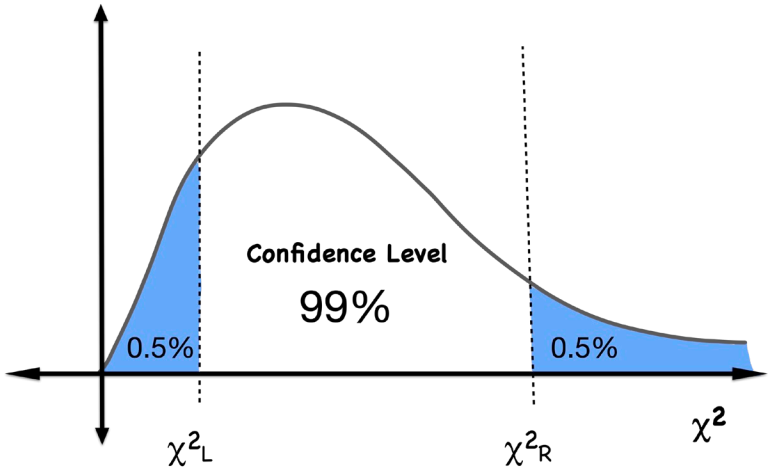
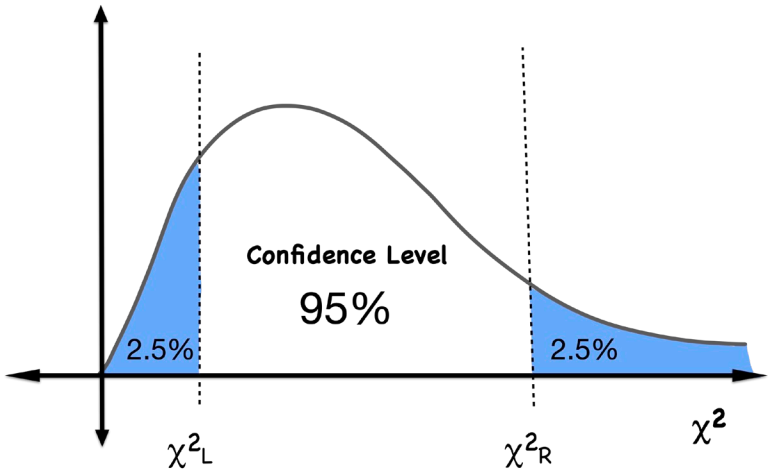
Estimate a True Standard Deviation

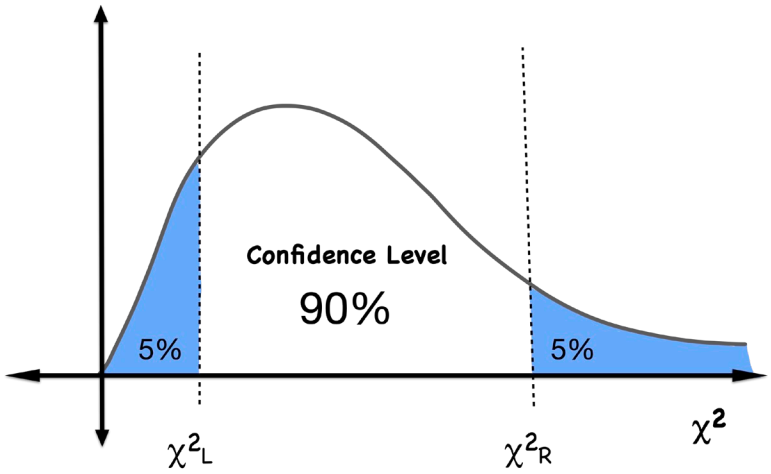
$$\sqrt{\frac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi_L^2}}$$

The Chi Square Distribution χ^2




Confidence Level
99%
0.5%
0.5%
χ^2_L
χ^2_R
χ^2


Confidence Level
95%
2.5%
2.5%
χ^2_L
χ^2_R
χ^2


Confidence Level
90%
5%
5%
χ^2_L
χ^2_R
χ^2

Formulas and Tables

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TABLE A-4 Chi-Square (χ^2) Distribution

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1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

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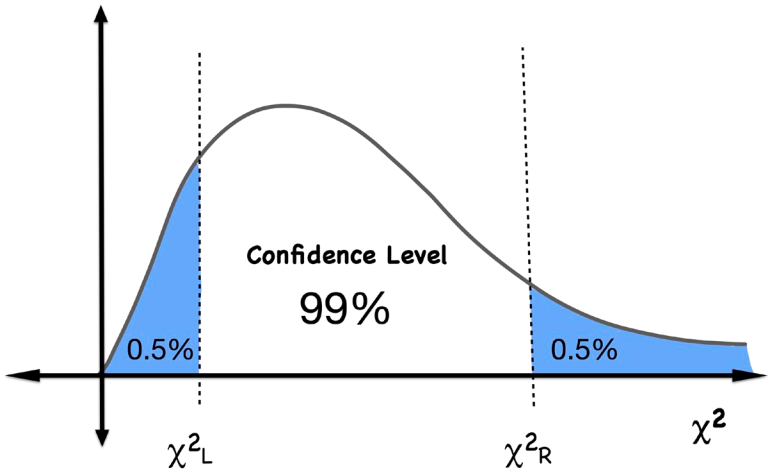
Years In College

A sample of 25 College students indicated it took them a mean of 6.8 years with a standard deviation of 0.9 years to earn a bachelor's degree.

1. Use the 99% confidence level to estimate the true variance in years it takes to earn a bachelor's degree.

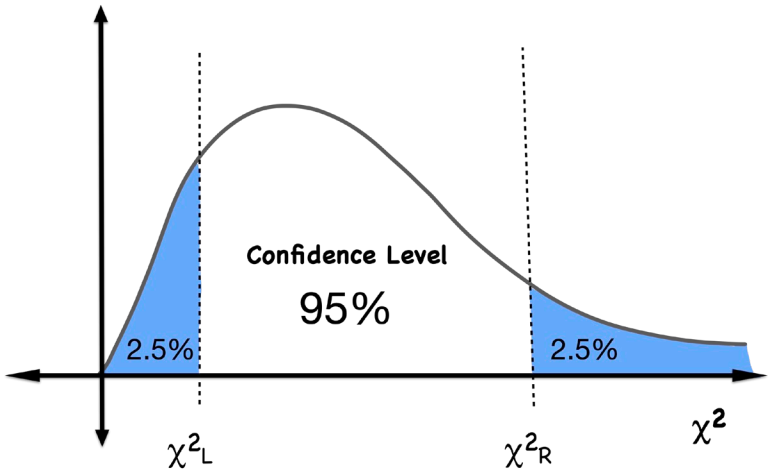
2. Use the 99% confidence level to estimate the true standard deviation in years it takes to earn a bachelor's degree.



Insomnia Treatment

A clinical trial was conducted to test the effectiveness of a drug on insomnia patients that had a mean wake time of 108.6 minutes without the drug. A sample of 18 insomnia patients who took the drug reported a mean wake time of 96.2 minutes with a standard deviation of 43.6 minutes.

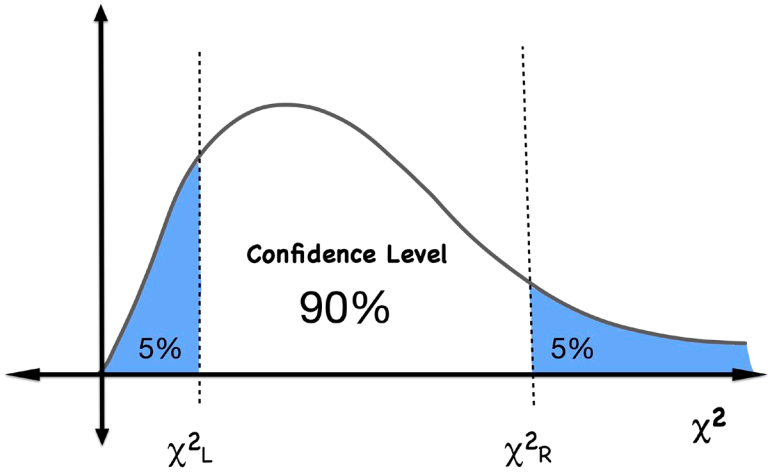
1. Use the 95% confidence level to estimate the true variance for wake time.
2. Use the 95% confidence level to estimate the standard deviation for wake time.



Age of Male College Students

A survey of 25 male college students report a mean age of 26.6 years with a standard deviation of 3.8 years.

1. Use the 90% confidence level to estimate the true variance for age of male college students.
2. Use the 90% confidence level to estimate the true standard deviation for age of male college students.



Estimating the Sample Size n

When conducting a new study to estimate a population parameter it is ideal to first determine the minimal sample size n needed for the study. In order to do that, you can use the following formula.

Estimate n

$$n = \frac{\left[z\alpha/_2\right]^2 \bar{p}\bar{q}}{E^2}$$ Proportion $\bar{p}$ is known

$$n = \frac{\left[z\alpha/_2\right]^2 0.25}{E^2}$$ Proportion $\bar{p}$ is unknown

$$n = \left(\frac{z\alpha/_2\,\sigma}{E}\right)^2$$ Mean

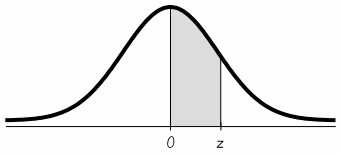


TABLE A-2 Standard Normal (z) Distribution

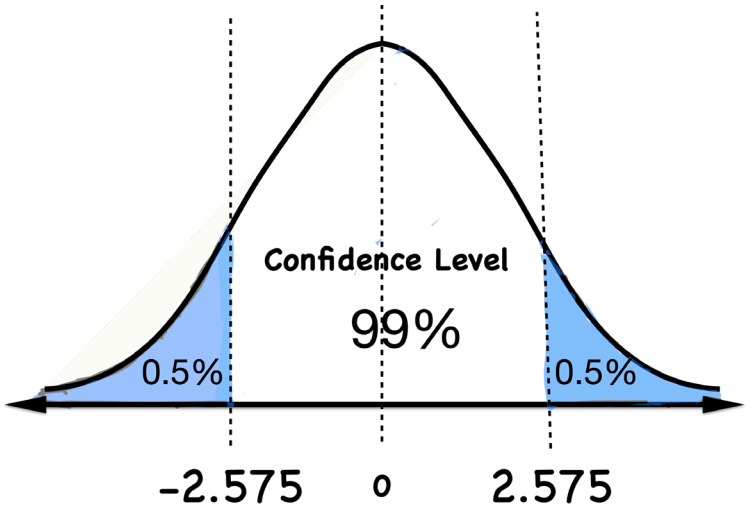
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495 *	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949 *	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.10 and higher	.4999									

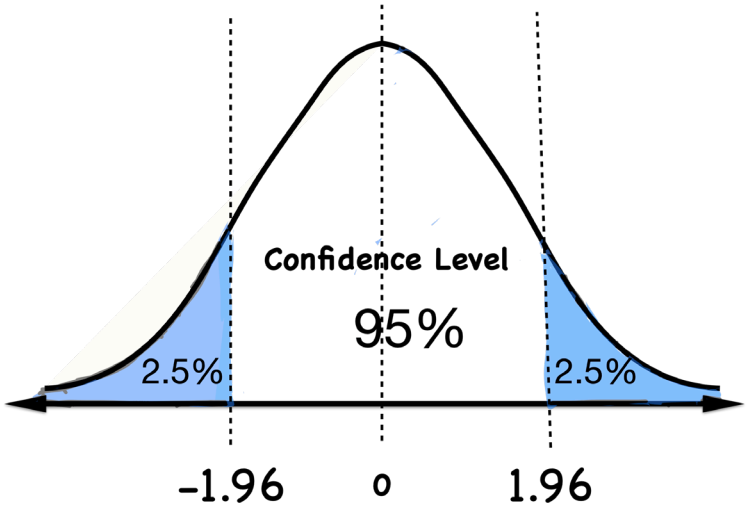
NOTE: For values of z above 3.09, use 0.4999 for the area.

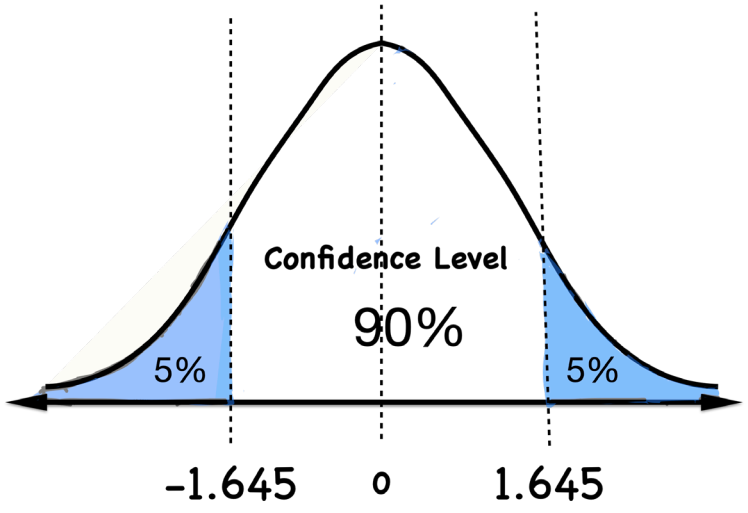
*Use these common values that result from interpolation:

z score	Area
1.645	0.4500
2.575	0.4950

From Frederick C. Mosteller and Robert E. K. Rourke, *Sturdy Statistics,* 1973, Addison-Wesley Publishing Co., Reading, MA. Reprinted with permission of Frederick Mosteller.


Confidence Level
99%
0.5%
0.5%
-2.575
0
2.575

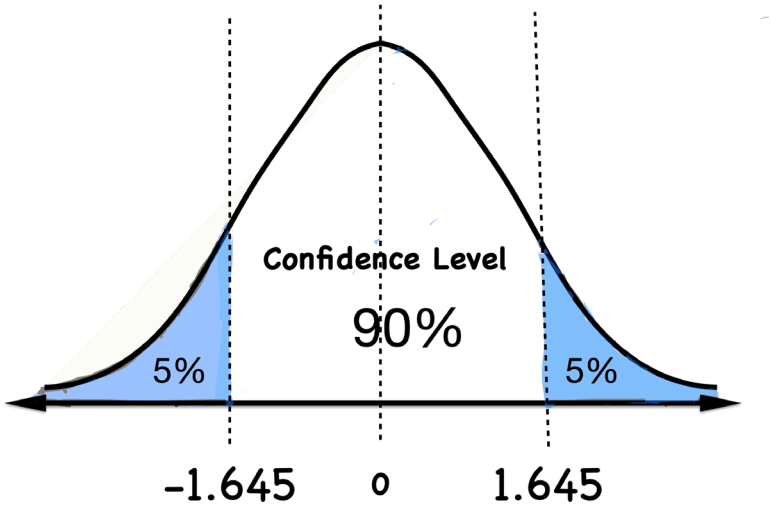

Confidence Level
95%
2.5%
2.5%
-1.96
0
1.96


Confidence Level
90%
5%
5%
-1.645
0
1.645

Lefties

A new study to estimate the true proportion of people who are left handed in California is going to be conducted using the 90% confidence level.

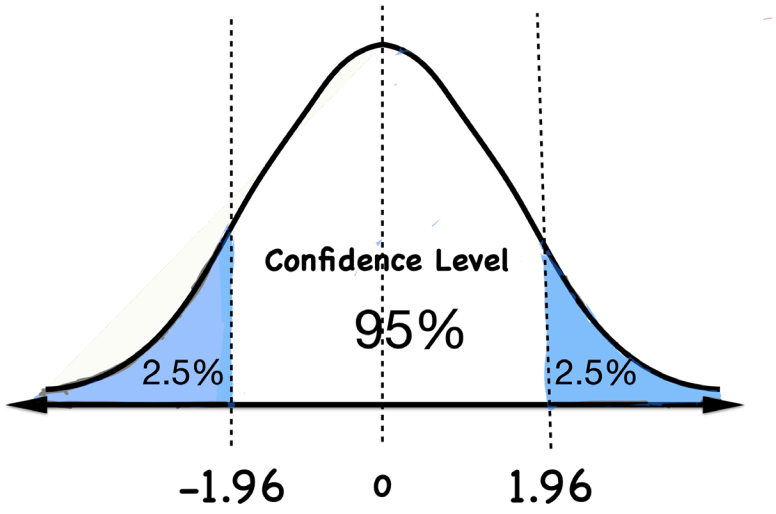
1. Estimate the sample size needed to be within a margin of error of 2%, if the sample proportion from a prior study is 9.5%.
2. Estimate the sample size needed to be within a margin of error of 2%, if the sample proportion from a prior study is unknown.



Social Media Gaming App

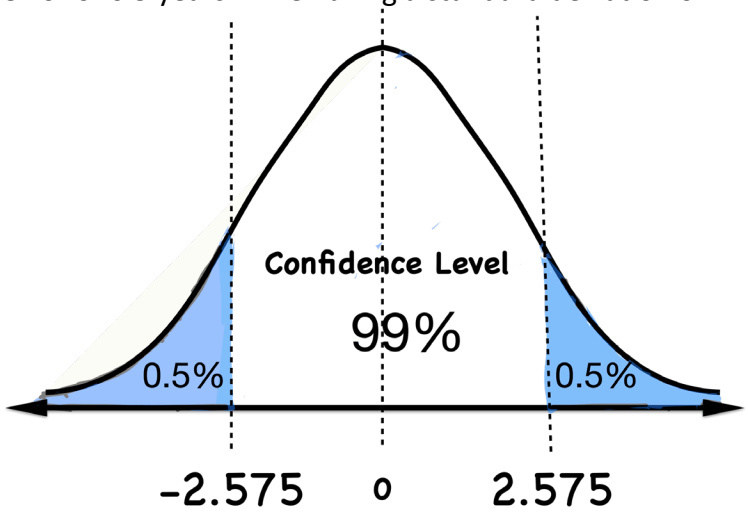
You are developing a new Social Media Gaming App to be advertised on Facebook and will be conducting a survey using the 95% confidence level to determine the true proportion that favor the app.

1. Estimate the sample size needed to be withing a margin of error of 3% for the new survey, if the past version of the Social Media Gaming App had a 52% favorable opinion.
2. Estimate the sample size needed to be withing a margin of error of 3% for the new survey, if the past version of the Social Media Gaming App is unknown.



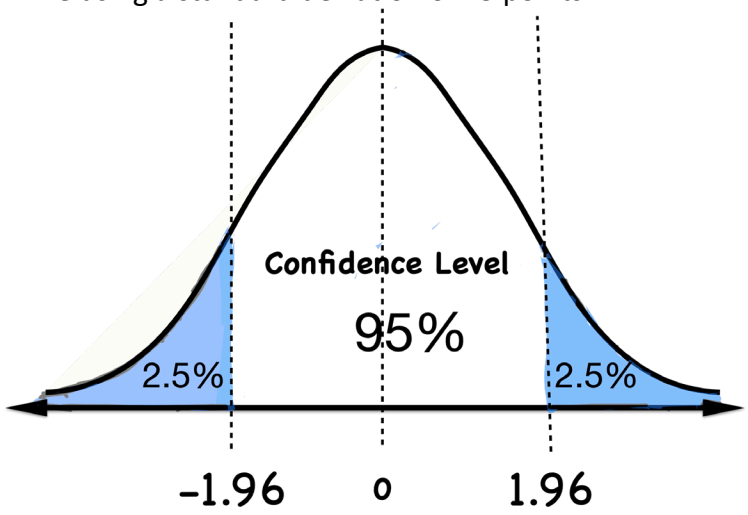
Female College Students

A new study on the mean age of female college students is to be conducted using the 99% confidence level. Estimate the sample size needed for the new study to be withing a margin of error of 0.5 years while having a standard deviation of 12.2 years from a past study.



IQ of College Professors

A new study on the mean IQ Score of College Professors is to be conducted using the 95% confidence level. Estimate the sample size needed to be withing a margin of error of 2.5 IQ points while using a standard deviation of 15 points.



Years In College

A new study on the mean number of years to obtain a Bachelor's Degree is to be conducted using the 90% confidence level. Estimate the sample size needed to be within a margin of error of 0.8 years while using a standard deviation of 1.8 years.