Volume of Revolution by Cylindrical Shells

Revolve a curve or region about the x-axis or y-axis to create a solid in order determine its volume

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Shell Method

Revolve the region bounded by the function $y = f(x)$ over $a \leq x \leq b$ about the y-axis

$$V = 2\pi \int_a^b rhdx = 2\pi \int_a^b xf(x)dx$$

Revolve the region bounded by the function $x = g(y)$ over $c \leq y \leq d$ about the x-axis

$$V = 2\pi \int_c^d rhdy = 2\pi \int_c^d yg(y)dy$$

Shell Method

Revolve the region bounded by the functions $y = f(x)$ and $y = g(x)$ over $a \leq x \leq b$ about the y-axis

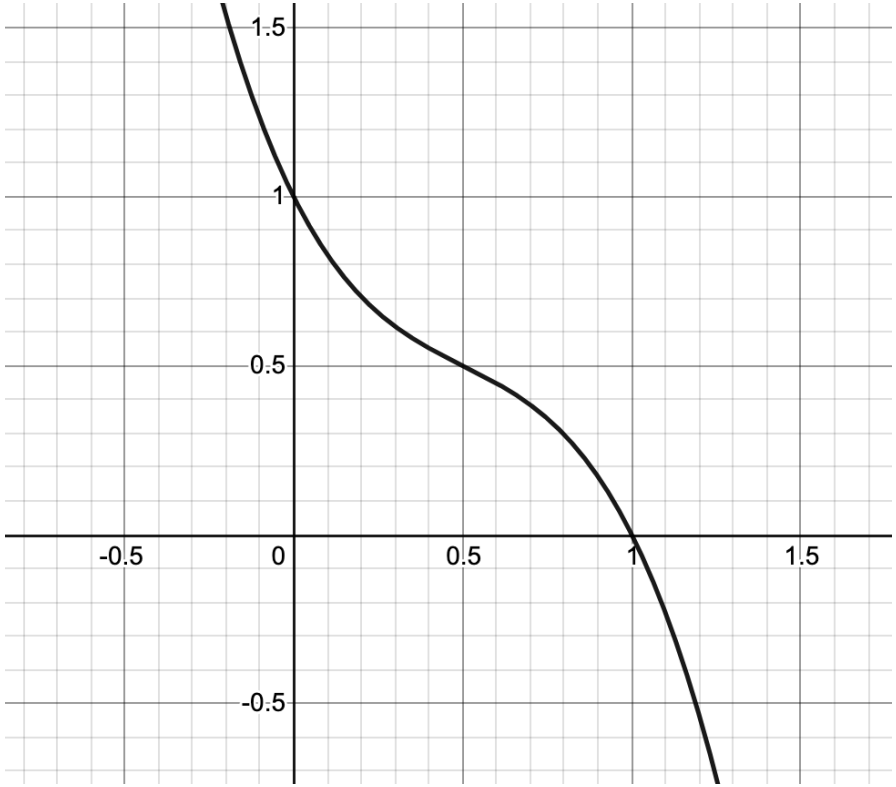
$$V = \int_a^b rhdx = 2\pi \int_a^b x[f(x) - g(x)]dx$$

Revolve the region bounded by the function $x = g(y)$ and $x = h(y)$ over $c \leq y \leq d$ about the x-axis

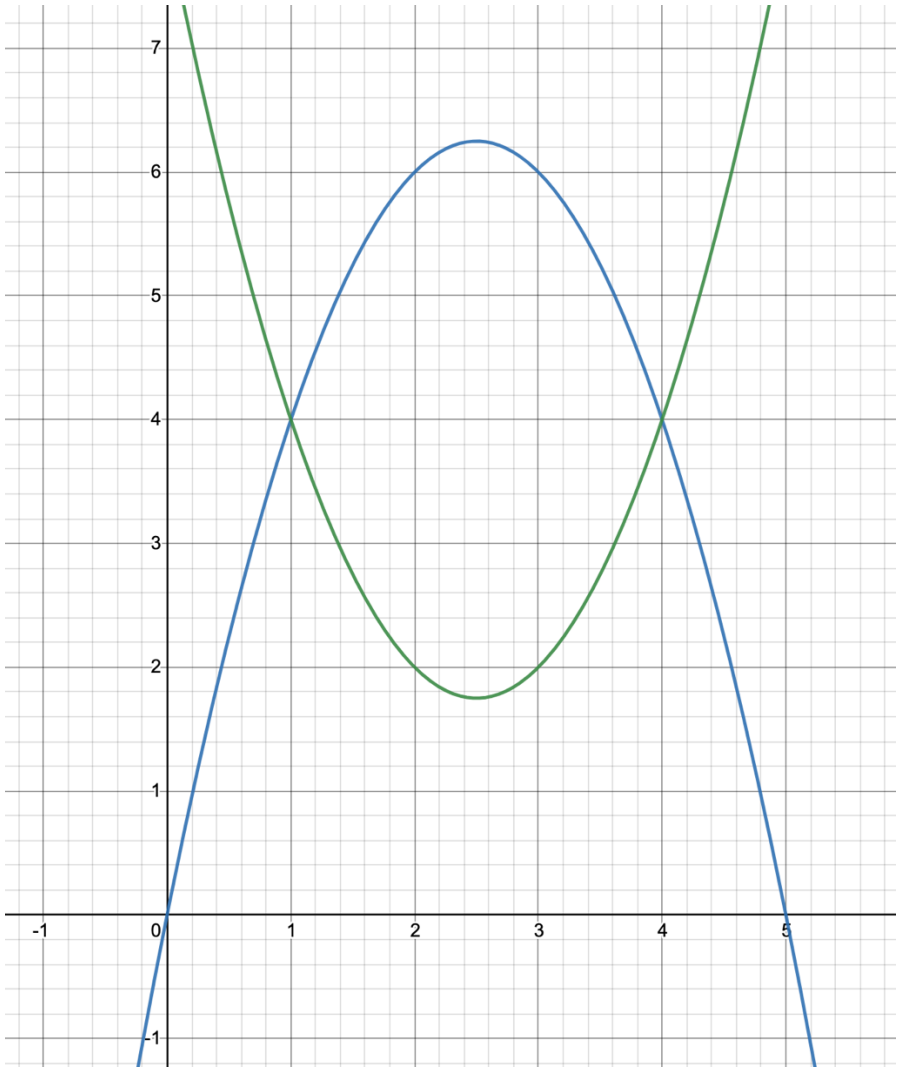
$$V = 2\pi \int_c^d rhdy = 2\pi \int_c^d y[g(y) - h(y)]dy$$

Determine the volume of the solid generated by rotating the following regions about the indicated axis.

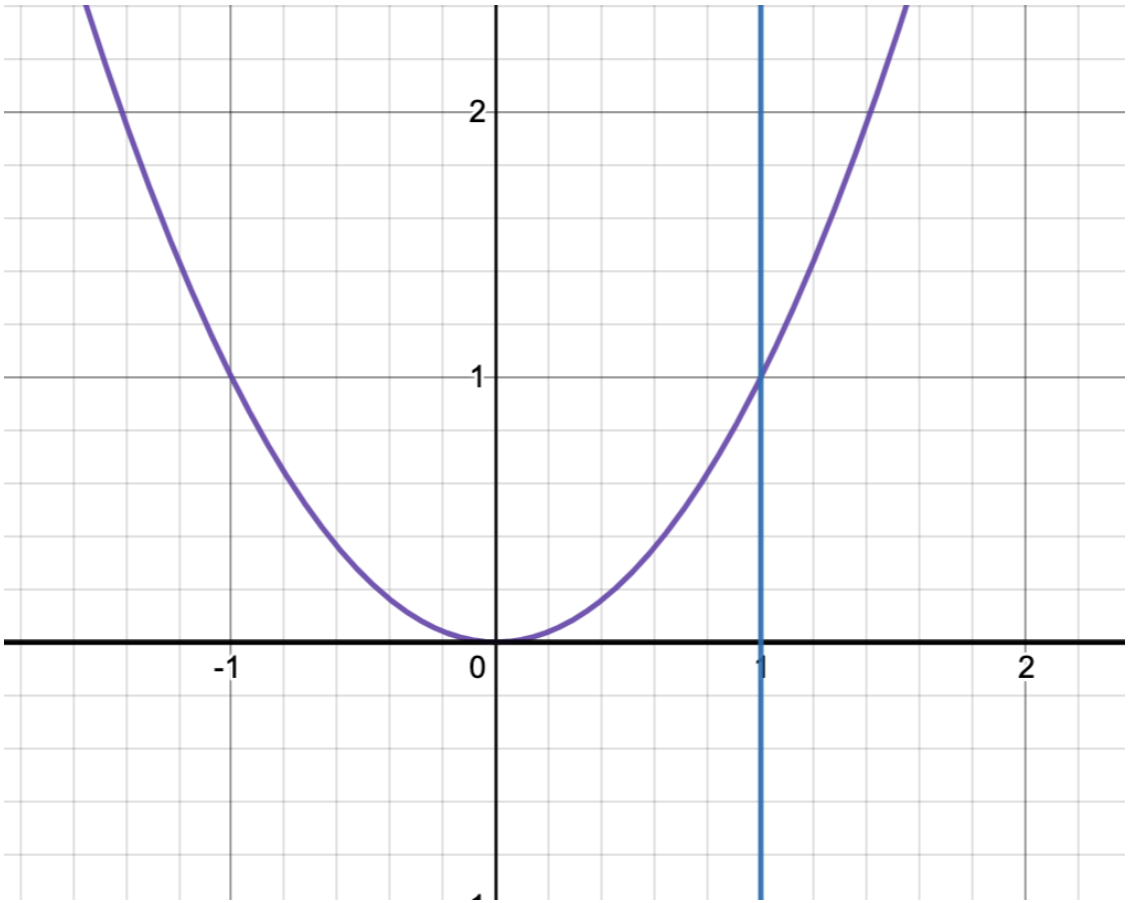
1. $f(x) = 1 - 2x + 3x^2 - 2x^3$ over $0 \leq x \leq 1$ rotate about the y-axis



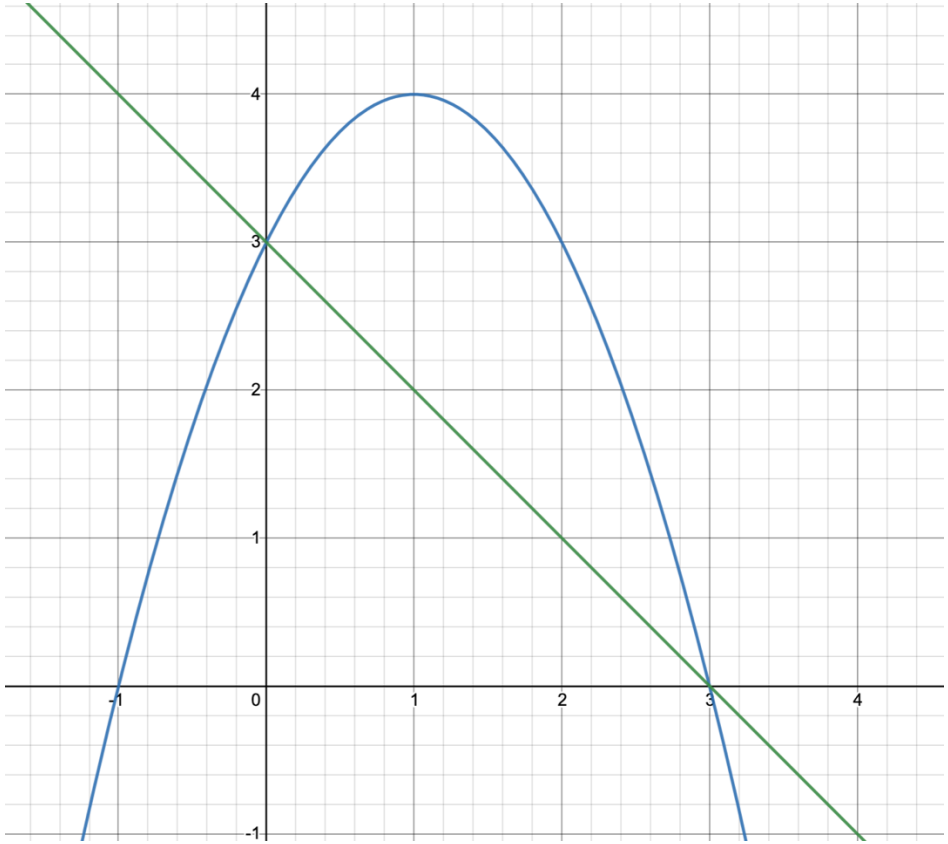
2. Region bounded by the curves $f(x) = x(5 - x)$ and $g(x) = 8 - x(5 - x)$ rotate about the y-axis



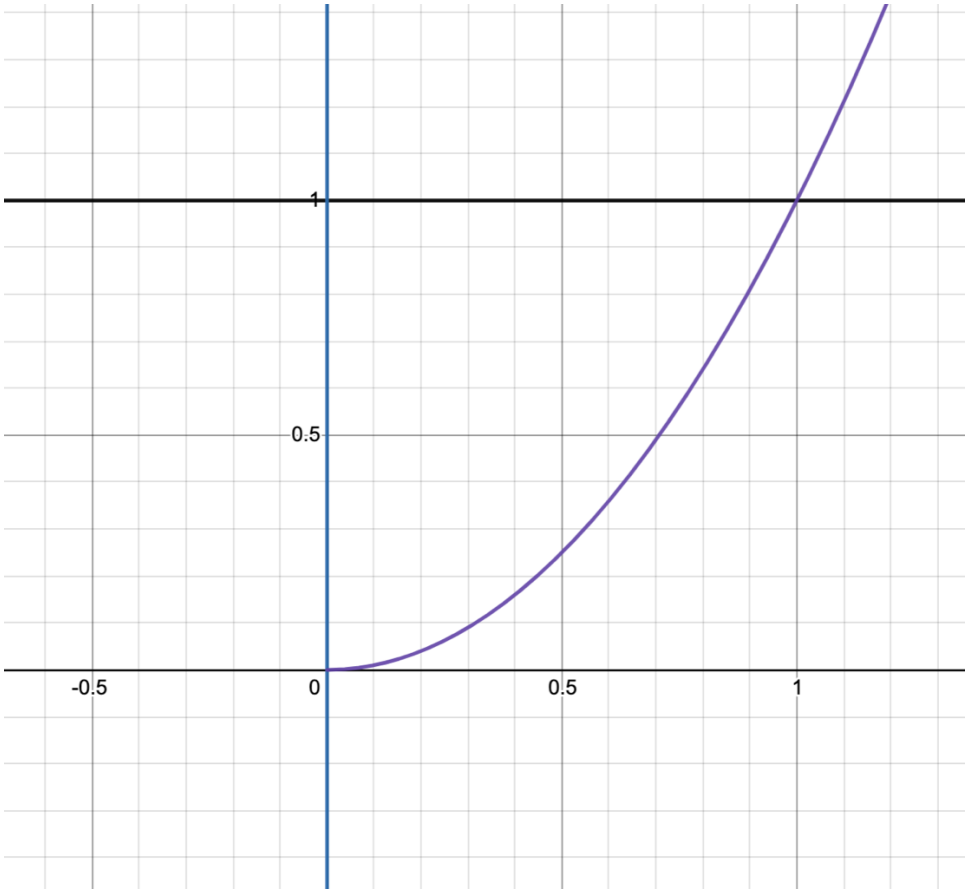
3. Region bounded by $y = x^2$, $y = 0$, $x = 1$ about the y-axis



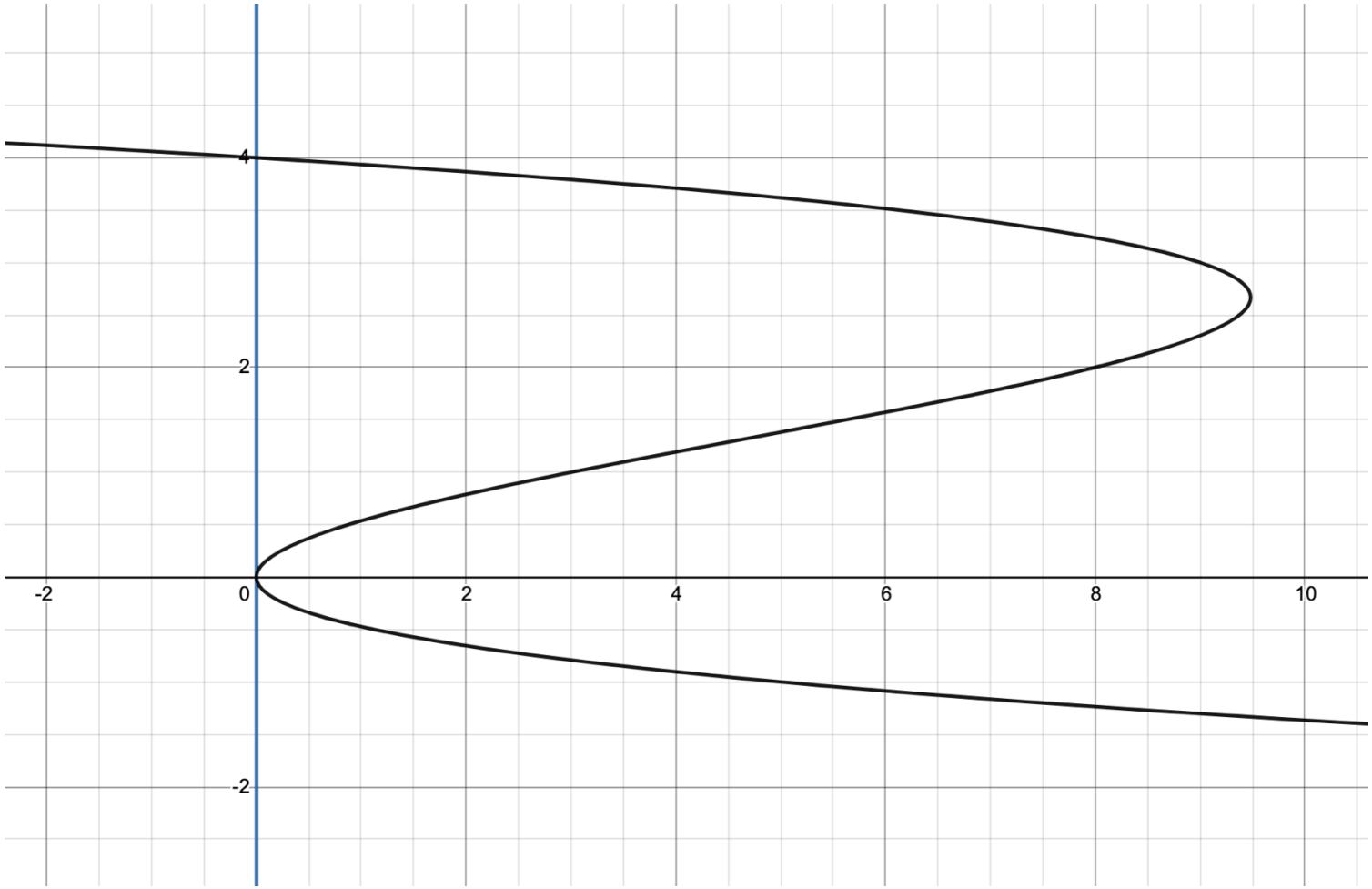
4. Region bounded by $y = 3 + 2x - x^2, x + y = 3$ about the y-axis



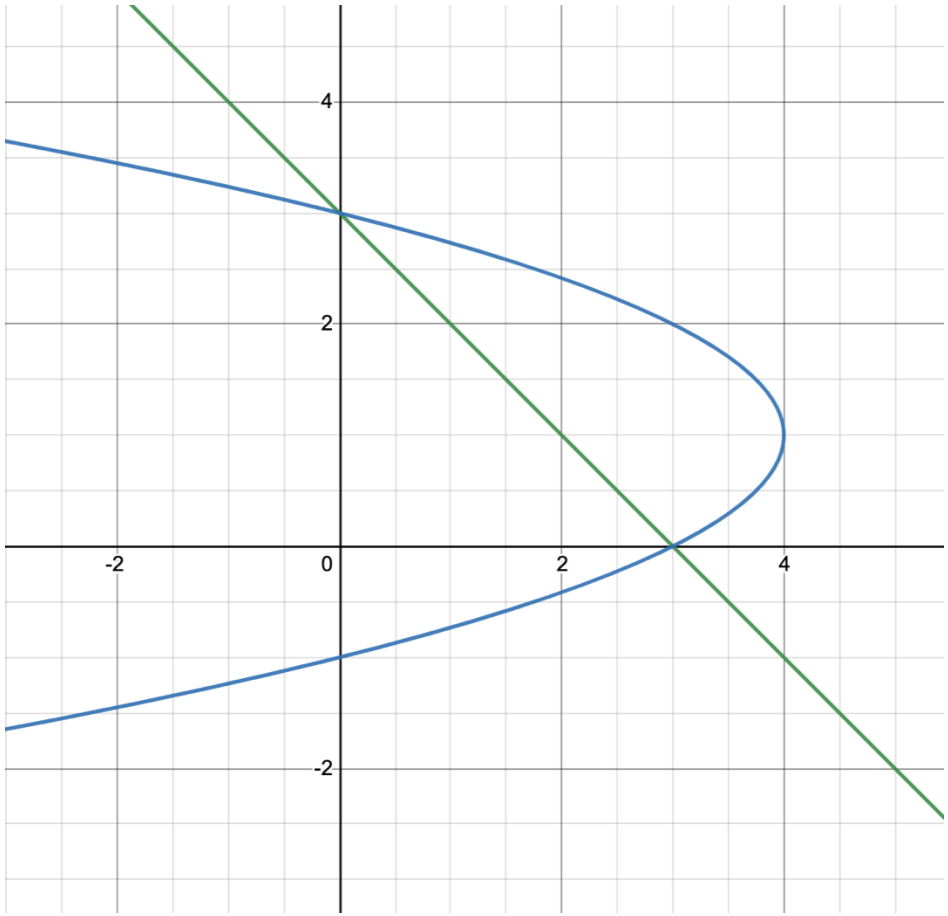
5. Region bounded by $x = \sqrt{y}, x = 0, y = 1$ about the x-axis



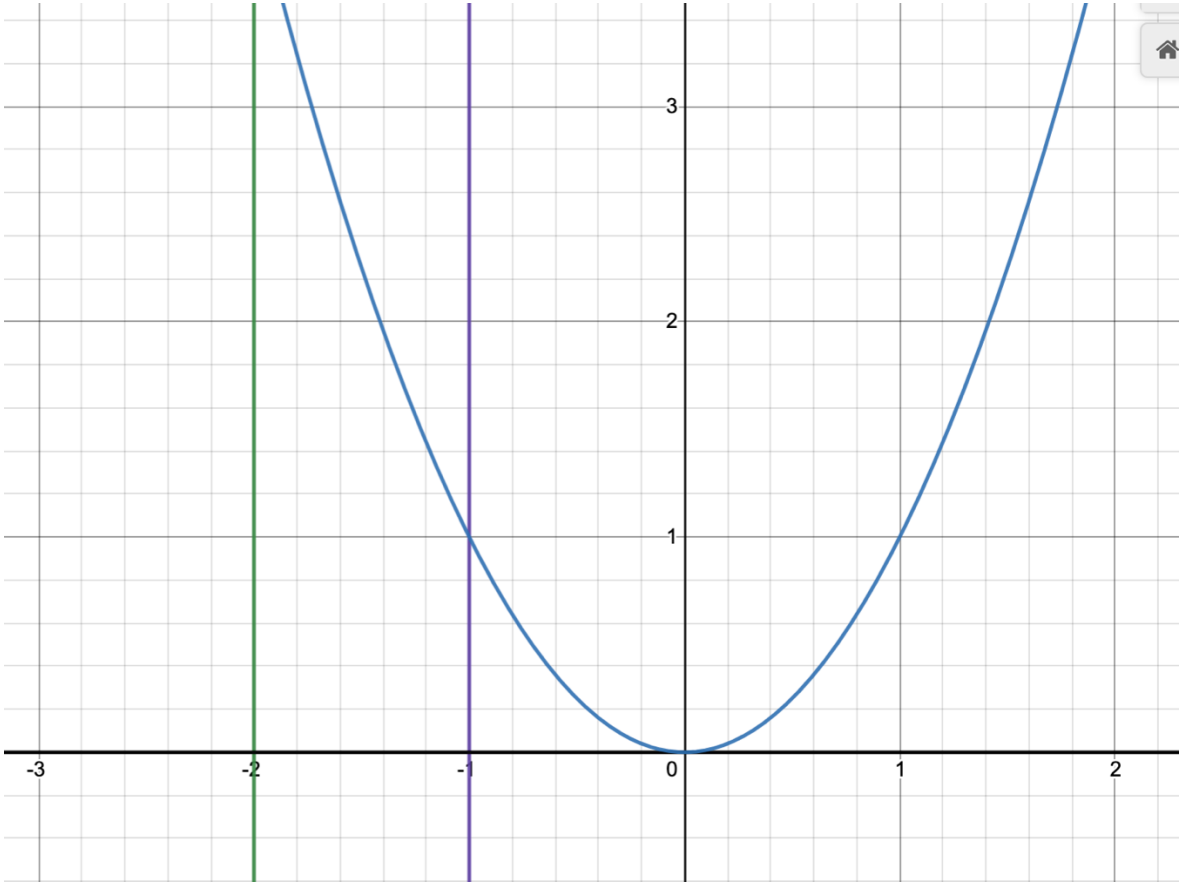
6. Region bounded by $x = 4y^2 - y^3,\ x = 0$ about the x-axis



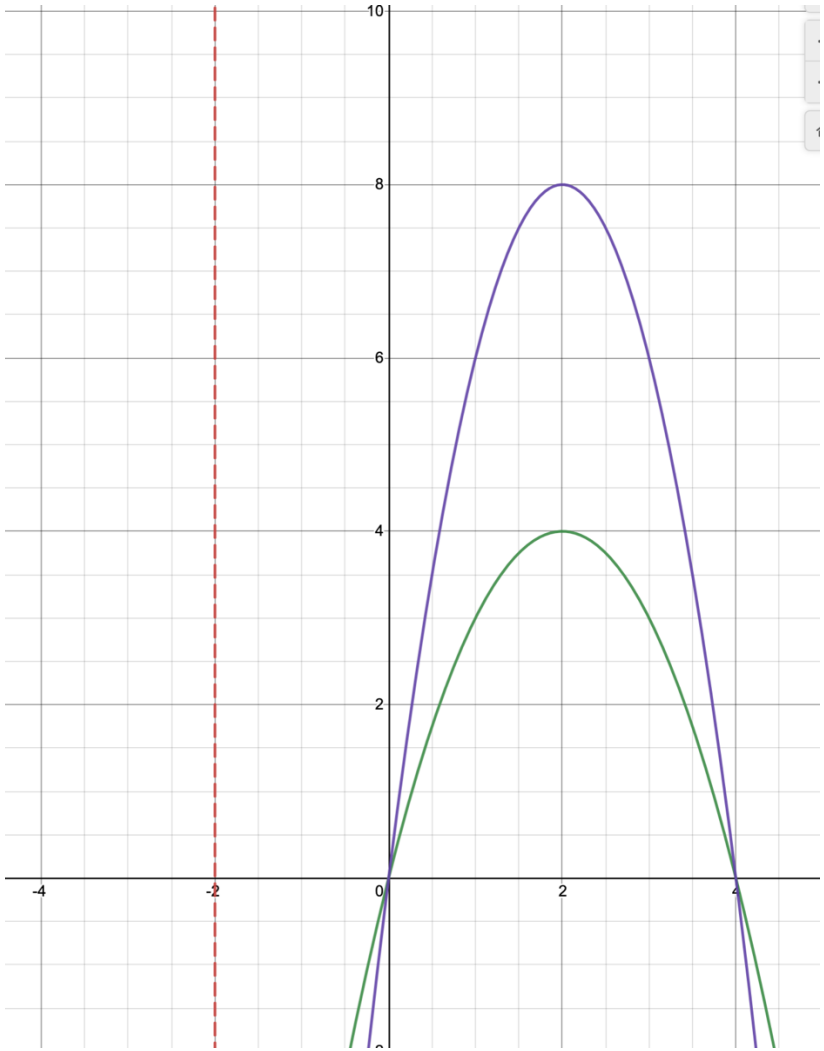
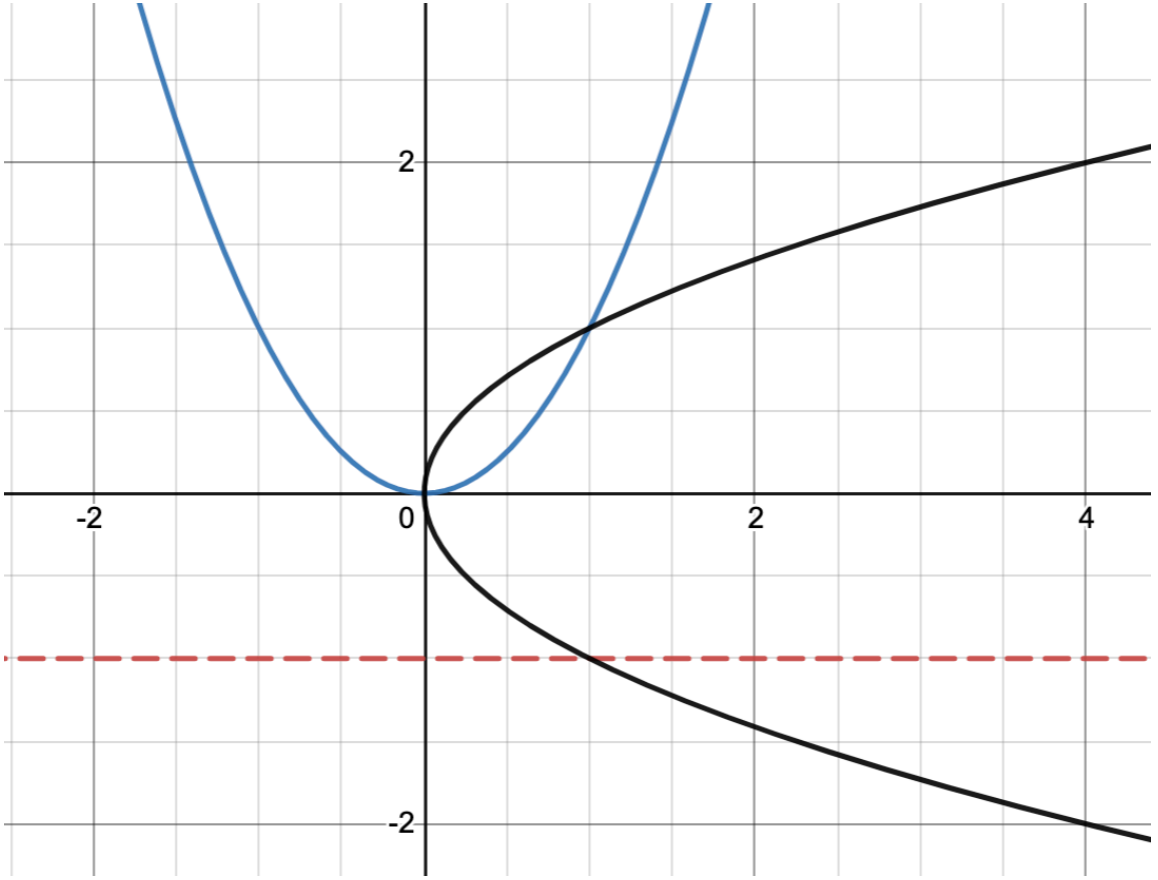
7. Region bounded by $x + y = 3, x = 4 - (y - 1)^2$ about the x-axis



8. Region bounded by $y = x^2, y = 0, x = -2, x = -1$ about y axis.



9. Region bounded by $y = 4x - x^2, y = 8x - 2x^2$ about $x = -2$



10. Region under the graph $y = x^2$, $x = y^2$ rotate about the line $y = -1$