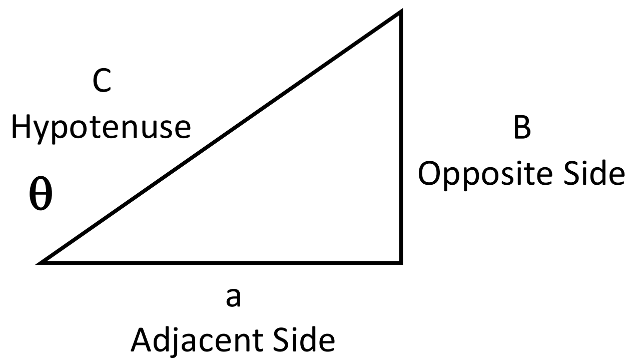
Trigonometric Substitution

We use Trigonometric Substitution for integrals that involve the following radicals. However, we will see that they can involve other functions as well.

$$\sqrt{a^2 - x^2}\ ,\ \sqrt{a^2 + x^2}, \sqrt{x^2 - a^2}$$

The tools we need are based on Trigonometry and we start with a right triangle with the following fundamental concepts.



Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Right Angle Definitions

$$sin(\theta) = \frac{b}{c}$$

$$cos(\theta) = \frac{a}{c}$$

$$tan(\theta) = \frac{b}{a}$$

Pythagorean Identities

$$sin^2(\theta) + cos^2(\theta) = 1$$
$$tan^2(\theta) + 1 = sec^2(\theta)$$
$$1 + cot^2(\theta) = csc^2(\theta)$$

Reciprocal Identities

$$sec(\theta) = \frac{1}{cos(\theta)}$$

$$csc(\theta) = \frac{1}{sin(\theta)}$$

$$cot(\theta) = \frac{1}{tan(\theta)}$$

These three types of integrals will involve similar procedures but different identites from Trigonometry.

Case 1

Integral that involves $\sqrt{a^2 - x^2}$

Substitution $x = asin(\theta)$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

Identity $1 - sin^2(\theta) = cos^2(\theta)$

Case 2

Integral that involves $\sqrt{a^2 + x^2}$

Substitution $x = atan(\theta)$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

Identity $1 + tan^2(\theta) = sec^2(\theta)$

Case 3

Integral that involves $\sqrt{x^2 - a^2}$

Substitution $x = asec(\theta)$ for $0 \leq \theta < \frac{\pi}{2}$ or $\pi \leq \theta < \frac{3\pi}{2}$

Identity $sec^2(\theta) - 1 = tan^2(\theta)$