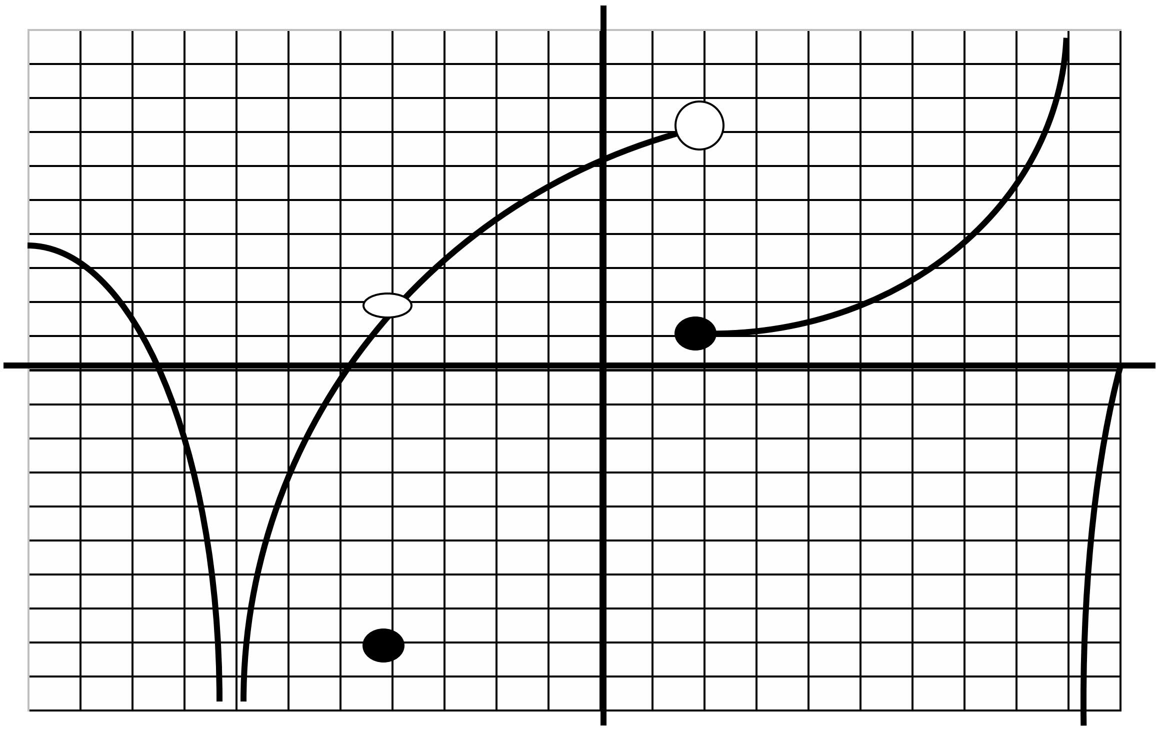
The Graphical Approach to Limits



Let's note the following **limits** by looking at the graph of some wildly exotic function f.

- $\lim_{x\to 2^-} f(x) = 7$ but $\lim_{x\to 2^+} f(x) = 1$, hence $\lim_{x\to 2} f(x) = DNE$
  Note- $f(2) = 1$

- $\lim_{x\to -4^-} f(x) = 2$ and $\lim_{x\to -4^+} f(x) = 2$, hence $\lim_{x\to -4} f(x) = 2$
  Note- $f(-4) = -8$

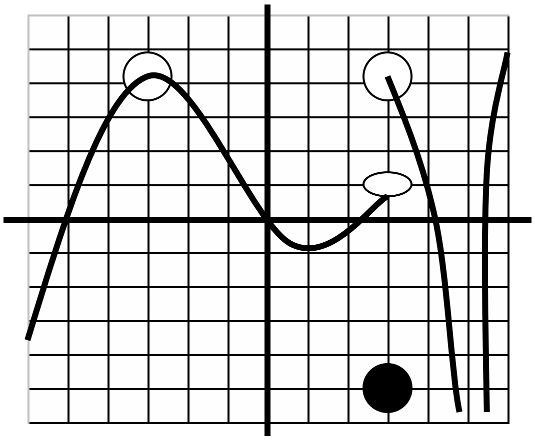
- $\lim_{x\to 7^-} f(x) = -\infty$ and $\lim_{x\to 7+} f(x) = -\infty$, hence $\lim_{x\to 7} f(x) = -\infty$
  Note- $f(-7) = undefined$

- $\lim_{x\to 9^-} f(x) = \infty$ but $\lim_{x\to 9^+} f(x) = -\infty$, hence $\lim_{x\to 9} f(x) = DNE$
  Note- $f(9) = undefined$

- $\lim_{x\to 5^-} f(x) = 2$ and $\lim_{x\to 5^+} f(x) = 2$, hence $\lim_{x\to 5} f(x) = 2$

This visual approach has its advantages; you don't have to tedious calculations. However, it's imperative that you are knowledgeable of graphs for various functions.

You try, Consider the following functions f and g; determine the following limits.



This is the graph of a function f.

$\lim_{x\to 3^-} f(x) =$

$\lim_{x\to 3^+} f(x) =$

$\lim_{x\to 0^-} f(x) =$

$\lim_{x\to 0^+} f(x) =$

$\lim_{x\to -3^-} f(x) =$

$\lim_{x\to -3^+} f(x) =$

$\lim_{x\to 5^-} f(x) =$

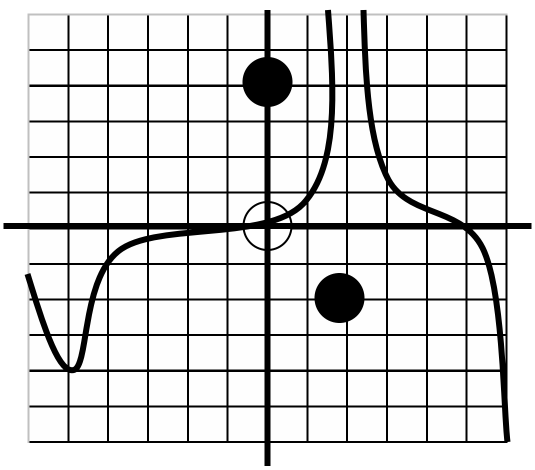
$\lim_{x\to 5^+} f(x) =$

$\lim_{x\to -5^-} f(x) =$

$\lim_{x\to -5^+} f(x) =$

Note the following values

f(3),f(0), f(-3), f(5), f(-5)



This is the graph of a function g.

$\lim_{x\to 2^-} g(x) =$

$\lim_{x\to 2^+} g(x) =$

$\lim_{x\to 0^-} g(x) =$

$\lim_{x\to 0^+} g(x) =$

$\lim_{x\to 3^-} g(x) =$

$\lim_{x\to 3^-} g(x) =$

$\lim_{x\to -5^-} g(x) =$

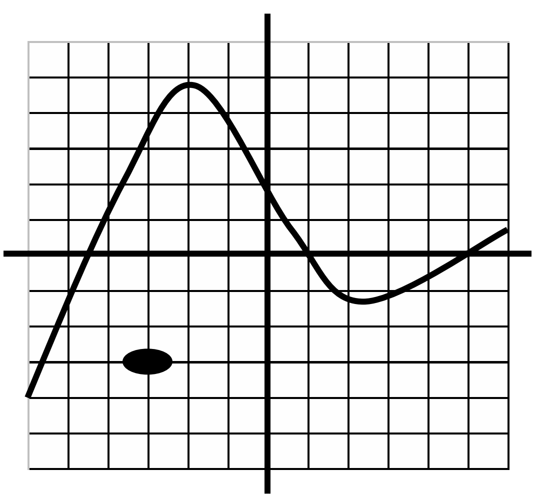
$\lim_{x\to -5^+} g(x) =$

$\lim_{x\to 5^-} g(x) =$

$\lim_{x\to 5^-} g(x) =$

Note the following values

g(2), g(0), g(3), g(-5), g(5)



$h(x) =$

This is the graph of a function h.

Properties of Limits

(Limit Arithmetic)

The following properties are also known as the limit laws. They describe what you are allowed to do regarding limits. We will save the proofs of these laws for another day.

nstant.

$= N$; where

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x a ; if n is even, we assume a>0.

Examples- Let f and g be two functions such that the following is true.
Let $\lim\limits_{x\to 0} f(x)=4$ and $\lim\limits_{x\to 0} g(x)=-2$ and $\lim\limits_{x\to 0} h(x)=5$ and use properties of limits to calculate the following.

- $\lim\limits_{x\to 0}[f(x)+g(x)]=\lim\limits_{x\to 0} f(x)+\lim\limits_{x\to 0} g(x)=4+(-2)=2$
- $\lim\limits_{x\to 0}[f(x)-g(x)]=\lim\limits_{x\to 0} f(x)-\lim\limits_{x\to 0} g(x)=4-(-2)=4+2=6$
- $\lim\limits_{x\to 0}[f(x)\cdot g(x)]=\lim\limits_{x\to 0} f(x)\cdot\lim\limits_{x\to 0} g(x)=4(-2)=-8$
- $\lim\limits_{x\to 0}\left[\dfrac{f(x)}{g(x)}\right]=\dfrac{\lim\limits_{x\to 0} f(x)}{\lim\limits_{x\to 0} g(x)}=\dfrac{4}{-2}=-2$
- $\lim\limits_{x\to 0}[f(x)]^3=\left[\lim\limits_{x\to 0} f(x)\right]^3=4^3=64$
- $\lim\limits_{x\to 0}\sqrt{f(x)}=\sqrt{\lim\limits_{x\to 0} f(x)}=\sqrt{4}=2$
- $\lim\limits_{x\to 0}[6\cdot g(x)]=6\cdot\lim\limits_{x\to 0} g(x)=6(-2)=-12$
- $$\begin{aligned}\lim_{x\to 0}[3f(x)+2g(x)-4h(x)]&=3\lim_{x\to 0} f(x)+2\lim_{x\to 0} g(x)-4\lim_{x\to 0} h(x)\\ &=3\cdot 4+2(-2)-4\cdot 5\\ &=12-4-20\\ &=-12\end{aligned}$$
- $$\begin{aligned}\lim_{x\to 0}\left[-5f(x)+g(x)\sqrt{h(x)}\right]&=-5\lim_{x\to 0} f(x)+\lim_{x\to 0} g(x)\sqrt{\lim_{x\to 0} h(x)}\\ &=-5\cdot 4+(-2)\sqrt{5}\\ &=-20-2\sqrt{5}\end{aligned}$$
- $$\lim_{x\to 0}\left[\frac{f(x)-3g(x)}{\sqrt{12-f(x)}}\right]=\frac{\lim\limits_{x\to 0}[f(x)-3g(x)]}{\lim\limits_{x\to 0}\sqrt{12-f(x)}}=\frac{\lim\limits_{x\to 0} f(x)-3\lim\limits_{x\to 0} g(x)}{\sqrt{12-\lim\limits_{x\to 0} f(x)}}$$
  $$=\frac{4-3(-2)}{\sqrt{12-4}}=\frac{4+12}{\sqrt{8}}=\frac{16}{\sqrt{8}}=\frac{16\sqrt{8}}{8}=2\sqrt{8}=4\sqrt{2}$$