Probability Distributions

Aka, Probability Formulas

The following is a frequency table describing the number of children College students have as described in lecture.

Number of Children	f	rf
0	12	0.375
1	8	0.25
2	4	0.125
3	5	0.156
4	2	0.063
5	1	0.031
Sum	32	1

What you didn't know at the time is that you were really looking at a probability distribution over a discrete random variable.

Def- Random Variable
A variable (letter) that typical holds a numerical value and is denoted by the letter x. This variable represents the different outcomes for an experiment.

Def- Discrete Random Variable
A random variable whose collection of values is finite or countable.

In the number of children example, we let $x = \#\, children$ where $x = 0,1,2,3,4,5$ and is an example of a discrete random variable.

Def- Continuous Random Variable
A random variable that has infinitely many values and the collections of values is not countable.

Fact- Every outcome in a random variable is associated with probabilities denoted by $P(x)$ and $\sum P(x) \approx 1$

Def- $P(x) = \text{The probability of a random variable x}$

It is preferred that we summarize these values in a table known as a **Probability Distribution**

$$x = \# \, children$$

Number of Children	f	rf
0	12	0.375
1	8	0.25
2	4	0.125
3	5	0.156
4	2	0.063
5	1	0.031
Sum	32	1

becomes

x	P(x)
0	0.375
1	0.25
2	0.125
3	0.156
4	0.063
5	0.031
Sum	1

Focus on the language of this probability distribution. If you select a college student at random, the probability the college student has:

1. No Children?

$P(x = 0) = 0.375$ shorter $P(0) = 0.375$

2. One Child?

$P(x = 1) = 0.25$ shorter $P(0) = 0.25$

3. Two Children?

$P(x = 2) = 0.125$ shorter $P(0) = 0.125$

4. Three Children?

$P(x = 3) = 0.156$ shorter $P(0) = 0.156$

5. Four Children?

$P(x = 4) = 0.063$ shorter $P(0) = 0.063$

6. Five Children?

$P(x = 5) = 0.031$ shorter $P(0) = 0.031$

Of course, the **language** we typically use in life and on tests is the following. If you select a College student at random, what's the probability the College student has:

7. At least One child?

$P(x \geq 1) = P(x = 1 \; or \; x = 2 \; or \; x = 3 \; or \; x = 4 \; or \; x = 5)$

$= P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5)$

$= P(1) + P(2) + P(3) + P(4) + P(5)$

$\approx 0.25 + 0.125 + 0.156 + 0.063 + 0.031$

≈ 0.625

8. At least two children?

$P(x \geq 2) = P(x = 2 \; or \; x = 3 \; or \; x = 4 \; or \; x = 5)$

$= \; P(x = 2) + P(x = 3) + P(x = 4) + P \; (x = 5)$

$= \; P(2) + P(3) + P(4) + P \; (5)$

$\approx 0.125 + 0.156 + 0.063 + 0.031$

≈ 0.375

9. More than two children?

$P(x > 2) = P(x = 3 \; or \; x = 4 \; or \; x = 5)$

$= P(x = 3) + P(x = 4) + P \; (x = 5)$

$= \; P(3) + P(4) + P \; (5)$

$\approx 0.156 + 0.063 + 0.031$

≈ 0.25

10. No more than three children?

$P(x \leq 3) = P(x = 1 \; or \; x = 2 \; or \; x = 3)$

$= P(x = 1) + P(x = 2) + P(x = 3)$

$= P(1) + P(2) + P(3)$

$\approx 0.25 + 0.125 + 0.156$

≈ 0.531

11. Less than three children?

$P(x < 3) = P(x = 1\ or\ x = 2)$

$= P(x = 1) + P(\ x = 2)$

$= P(1) + P(2)$

$\approx 0.25 + 0.125$

≈ 0.15

12. Between one and four children?

$P(1 \leq x \leq 4) = P(x = 1\ or\ x = 2\ or\ x = 3\ or\ x = 4)$

$= P(x = 1) + P(\ x = 2) + P(\ x = 3) + P(\ x = 4)$

$= P(1) + P(\ 2) + P(\ 3) + P(\ 4)$

$\approx 0.25 + 0.125 + 0.156 + 0.063$

≈ 0.594

Fact- The Complement Rule for Probability is going to extremely helpful for some of these questions we answered, so stay tuned for lecture.

$$P(E) + P(\bar{E}) = 1$$

We can even compute the mean, variance, and standard deviation for any probability distribution.

Mean $\mu = \sum_{all\ x} xP(x)$

Variance $\sigma^2 = \sum_{allx} x^2 P(x) - \mu^2$

Standard Deviation $\sigma = \sqrt{\sum_{allx} x^2 P(x) - \mu^2}$

Def- Expected Value for a Distribution

$$\mu = \sum_{all\ x} xP(x)$$

If you notice that this is the same definition as the mean, you are correct! This definition is also known as the long run average or the expectation. We will see many uses of it in lecture.

Raffle

ELAC is raffling off an iPad Pro ($ 1,200 value) and has sold 2,500 tickets. If the cost of a ticket is $2.00, compute the expected value for this raffle.

Outcome	Amount	Probability
Win		
Lose		

Three Aces
Las Vegas is offering a new game for people to play and all you have to do is selecting three different aces without replacement. If it cost's $5.00 for a chance to win $1,000.00, compute the expected value for this game.

Outcome	Amount	Probability
Win		
Lose		

Life Insurance
A $ 35,000 life insurance policy cost's $ 250 per year for a 32-year old male. If the probability a 32-year old male lives to see the next year is 0.975, compute the expected value.

Outcome	Amount	Probability
Live		
Don't Live		