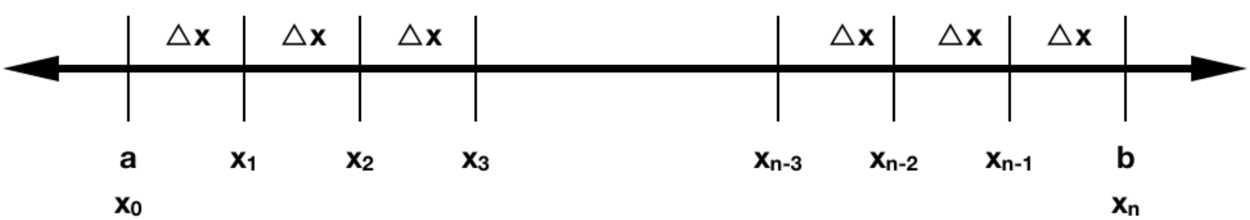
Numerical Approximation to Integration

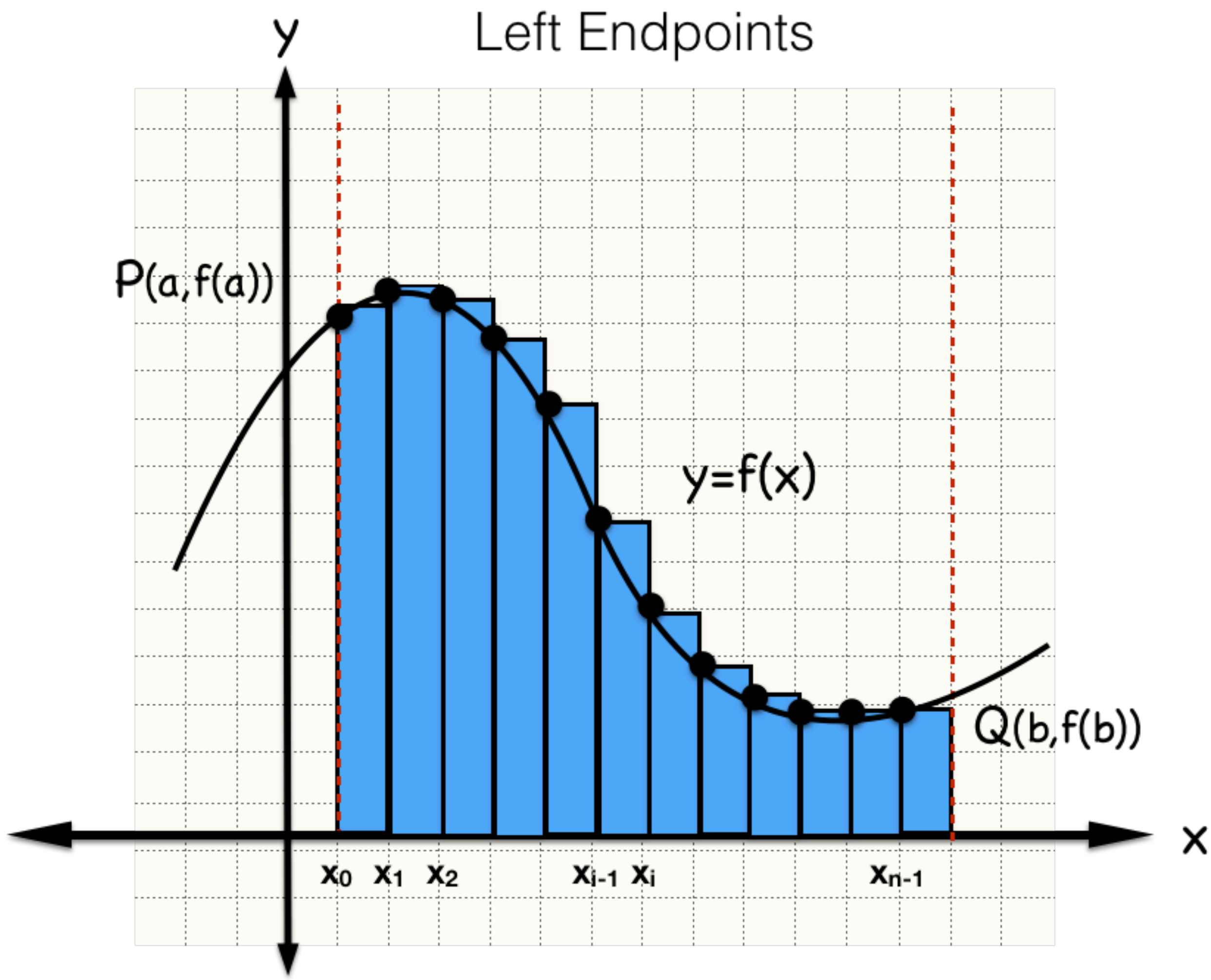
$$\int_a^b f(x)dx \approx formula$$

- Let f be a continuous function over a closed interval $[a, b]$ and chose the number of partitions n.
- Partition the interval $[a, b]$ into n-subintervals of equal length Δx where $\Delta x = \frac{b-a}{n}$

Left Endpoint Approximation



$$\int_a^b f(x)dx \approx L_n$$



where

$L_n = \sum_{i=0}^{n-1} f(x_i)\Delta x$

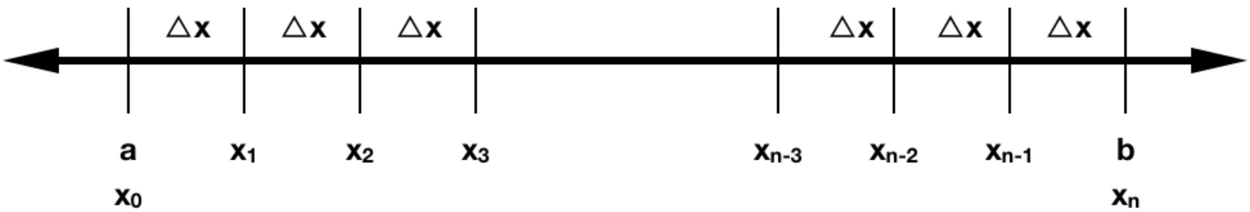
$L_n = f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_{n-2})\Delta x + f(x_{n-1})\Delta x$

$L_n = [f(x_0) + f(x_1) + f(x_2) + \cdots + f(x_{n-2}) + f(x_{n-1})]\Delta x$

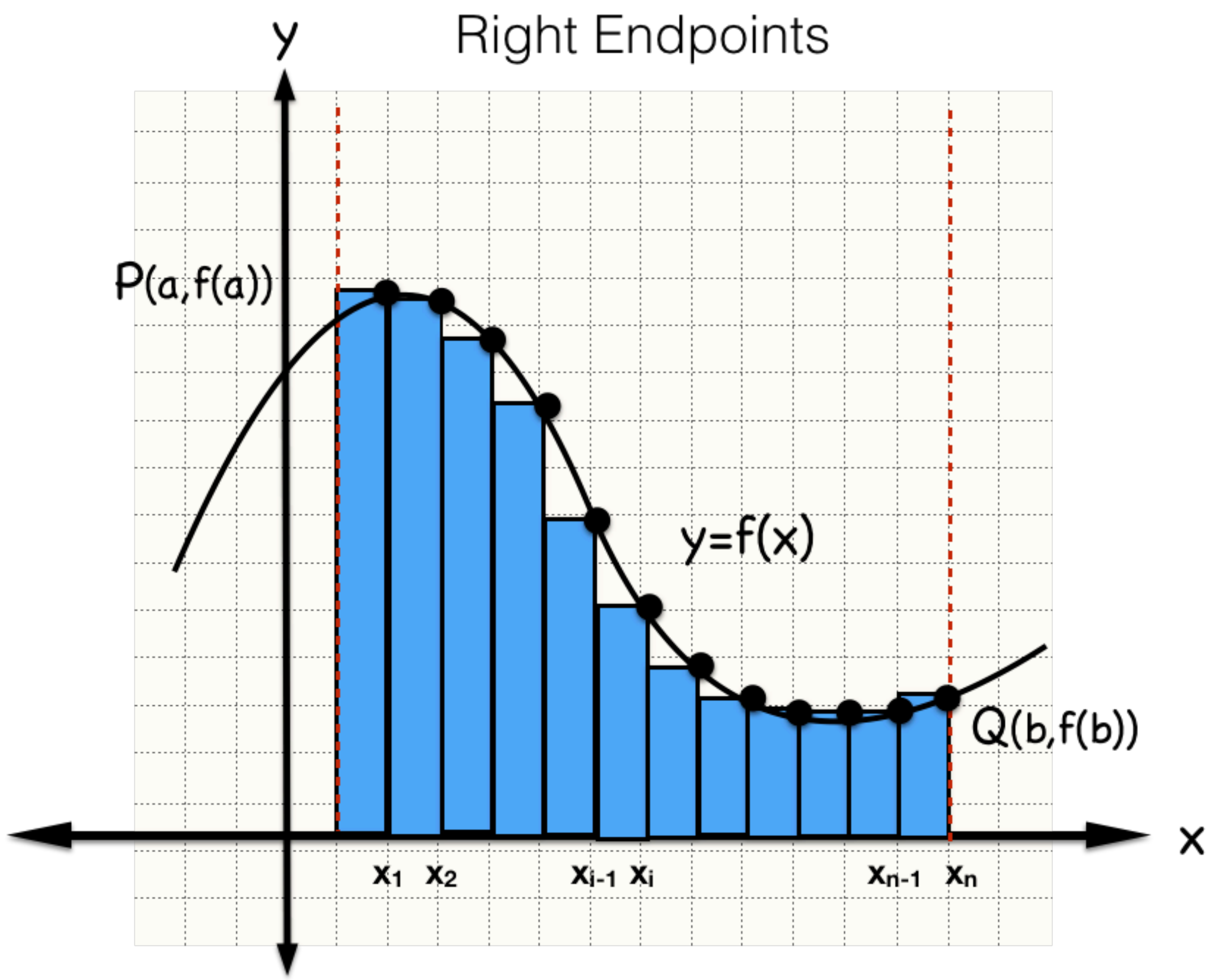
That is,

$$\int_a^b f(x)dx \approx \sum_{i=0}^{n-1} f(x_i)\Delta x$$

Right Endpoint Approximation



$$\int_a^b f(x)dx \approx R_n$$



where

$R_n = \sum_{i=1}^{n} f(x_i)\Delta x$

$R_n = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + \cdots + f(x_{n-1})\Delta x + f(x_n)\Delta x$

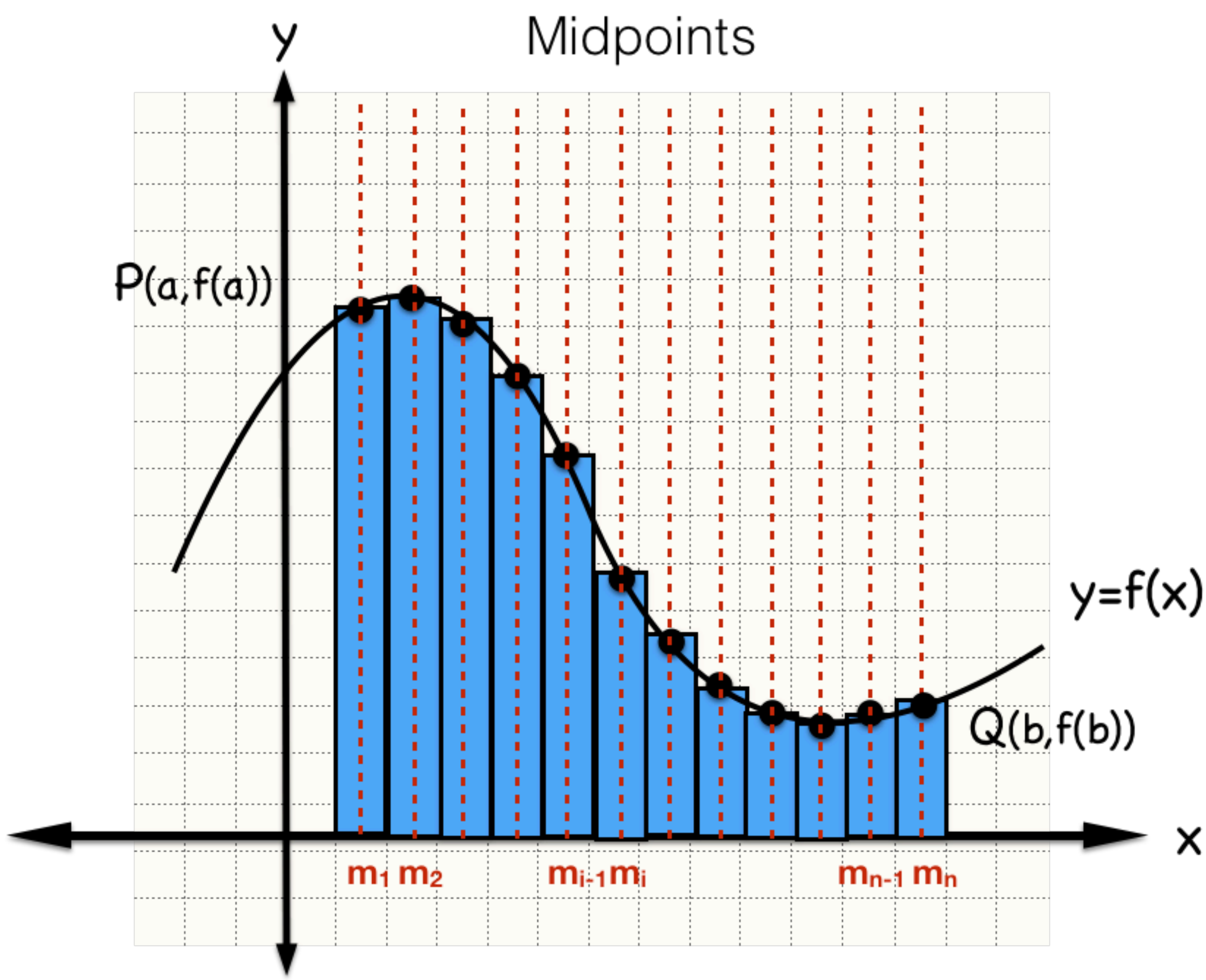
$R_n = [f(x_1) + f(x_2) + f(x_3) + \cdots + f(x_{n-1}) + f(x_n)]\Delta x$

That is,

$$\int_a^b f(x)dx \approx \sum_{i=1}^{n} f(x_i)\Delta x$$

Midpoint Approximation

$$\int_a^b f(x)dx \approx M_n$$



where $m_i = \frac{x_{i-1}+x_i}{2}$ is the midpoint of the interval $[x_{i-1}, x_i]$ from $i = 1\ to\ n$

$M_n = \sum_{i=1}^{n} f(m_i)\Delta x$

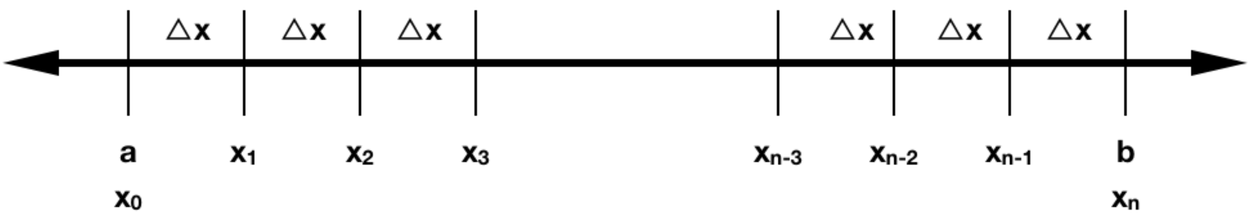
$M_n = f(m_1)\Delta x + f(m_2)\Delta x + f(m_3)\Delta x + \cdots + f(m_{n-1})\Delta x + f(m_n)\Delta x$

$M_n = [f(m_1) + f(m_2) + f(m_3) + \cdots + f(m_{n-1}) + f(m_n)]\Delta x$

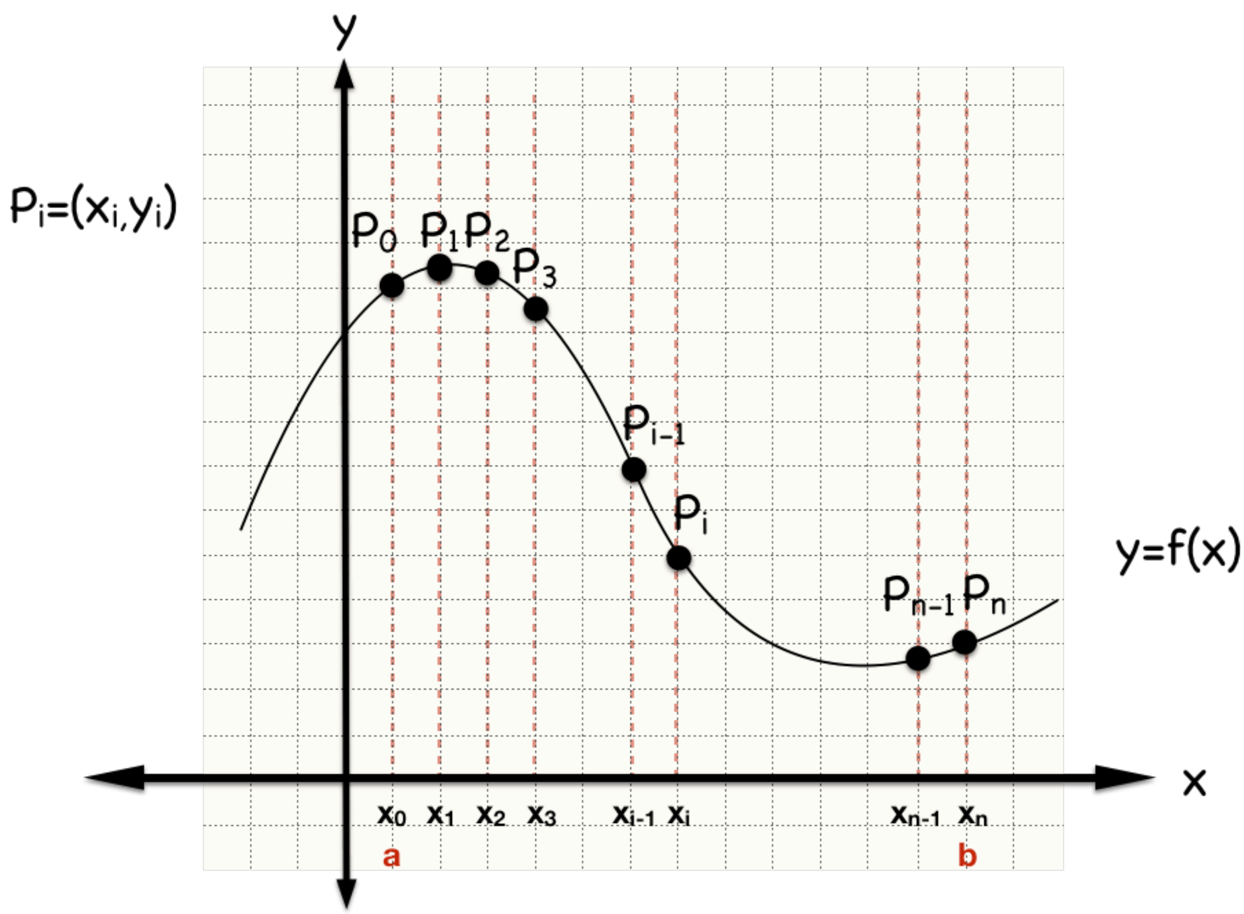
That is,

$$\int_a^b f(x)dx \approx \sum_{i=1}^{n} f(m_i)\Delta x$$

Simpson's Rule Approximation



$$\int_a^b f(x)dx \approx S_n$$



We can numerically approximate a definite integral over a closed interval and a continuous function. We require an even number of partitions

Let n (even) the number of partitions and $\Delta x = \frac{b-a}{n}$

Then

$$\boldsymbol{S_n = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]}$$

That is,

$$\boldsymbol{\int_a^b f(x)dx \approx \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]}$$

Note The weights to the function is of the sequence **1,4,2,4,2,…,2,4,1**