East Los Angeles College
Department of Mathematics
Math 241
Test 4

Solutions

Verify the identity.

1. $sin(2x) = 2sin(x)cos(x)$

$LHS = RHS$

$LHS = \sin(2x)$

$= \sin(x+x)$

$= \sin x \cos x + \cos x \sin x$

$= 2\sin x \cos x$

$= RHS$

2. $cos(2x) = 1 - 2sin^2(x)$

$LHS = RHS$

5 $LHS = \cos(2x)$

$= \cos(x+x)$

$= \cos x \cos x - \sin x \sin x$

$= \cos^2 x - \sin^2 x$

$= 1 - \sin^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

$= RHS$

Write in terms of sine only.

5. $y = -\sqrt{3}\sin x + 1\cos x$

A B

$$\cos\phi = \frac{-\sqrt{3}}{2}$$

$$\sin\phi = \frac{1}{2}$$

$$k = \sqrt{A^2 + B^2}$$

$$k = \sqrt{(-\sqrt{3})^2 + 1^2}$$

$$k = \sqrt{3+1}$$

$$k = \sqrt{4}$$

$$k = 2$$

$$\phi = \frac{\pi}{6}$$

$$\phi = \pi - \frac{\pi}{6} \; ; \; \phi = \frac{5\pi}{6}$$

$$\boxed{y = 2\sin\left(x + \frac{5\pi}{6}\right)}$$

Use formulas for lowering to a single power.

6. $sin^2(x)cos^2(x)$

$$\frac{1}{2}[1 - \cos(2x)]\,\frac{1}{2}[1 + \cos(2x)]$$

$$\frac{1}{4}[1 - \cos(2x)][1 + \cos(2x)]$$

$$\frac{1}{4}[1 - \cos^2(2x)]$$

$$\frac{1}{4} - \frac{1}{4}\cos^2(2x)$$

$$\frac{1 + \cos(4x)}{2}$$

$$\frac{1}{4} - \frac{1}{4}\cdot\frac{1}{2}[1 + \cos(4x)]$$

$$\frac{1}{4} - \frac{1}{8}[1 + \cos(4x)]$$

$$\frac{1}{4} - \frac{1}{8} - \frac{1}{8}\cos(4x)$$

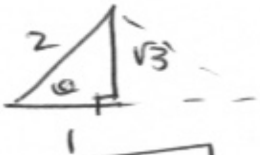
$$\frac{2}{8} - \frac{1}{8} - \frac{1}{8}\cos(4x)$$

$$\boxed{\frac{1}{8} - \frac{1}{8}\cos(4x)}$$

Determine the exact value without using a calculator.

7. $\cos^{-1}\left(\frac{1}{2}\right) = \theta$

$\cos\theta = \frac{1}{2}\ \frac{adj}{hyp}$

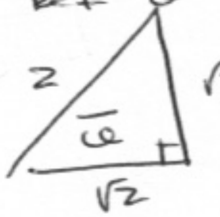


$\boxed{\theta = \frac{\pi}{3}}$

8. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \theta$

$\sin\theta = -\frac{\sqrt{2}}{2}$

Ref $\bar{\theta}$; $\sin\bar{\theta} = \frac{\sqrt{2}}{2}\ \frac{opp}{hyp}$



$\bar{\theta} = \frac{\pi}{4}$

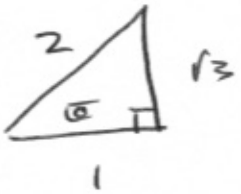
i.e, $\boxed{\theta = -\frac{\pi}{4}}$

9. $\tan^{-1}(-\sqrt{3}) = \theta$

$\tan\theta = -\sqrt{3}$

Ref $\bar{\theta}$

$\tan\bar{\theta} = \frac{\sqrt{3}}{1}\ \frac{opp}{adj}$



$\bar{\theta} = \frac{\pi}{3}$

i.e, $\boxed{\theta = -\frac{\pi}{3}}$

10. $\cos^{-1}(0) = \theta$

$\cos\theta = 0$

$\boxed{\theta = \frac{\pi}{2}}$

Solve the following trigonometric equations for x.

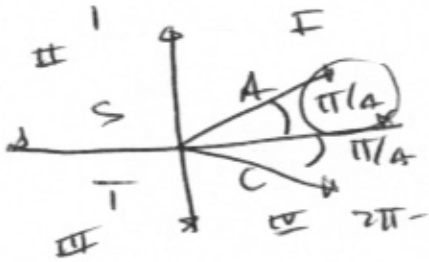
11. $\sqrt{2}\cos(x) - 1 = 0$

$\sqrt{2}\cos x = 1$

$\cos x = \frac{1}{\sqrt{2}}$ $\frac{opp}{hyp}$

$$\boxed{\begin{aligned} x_1 &= \frac{\pi}{4} + 2n\pi \\ x_2 &= \frac{7\pi}{4} + 2n\pi \end{aligned}}$$

$x = \frac{\pi}{4}$

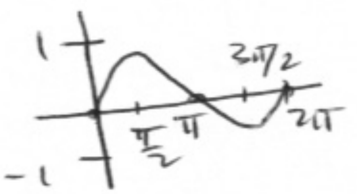


$2\pi - \frac{\pi}{4}$ $\frac{7\pi}{4}$

12. $(\sin(x) - 1)(\tan(x) + \sqrt{3}) = 0$

$\sin x - 1 = 0$

$\sin x = 1$

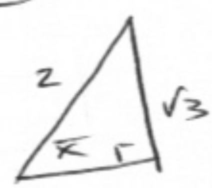


$x = \frac{\pi}{2}$

$\tan x + \sqrt{3} = 0$

$\tan x = -\sqrt{3}$

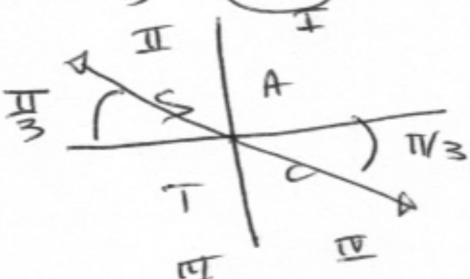
Rf $\tan \bar{x} = \frac{\sqrt{3}}{1}$



$\bar{x} = \frac{\pi}{3}$

$$\boxed{\begin{aligned} x_1 &= \frac{\pi}{2} + 2n\pi \\ x_2 &= \frac{2\pi}{3} + n\pi \end{aligned}}$$

$\pi - \frac{\pi}{3}$ $2\pi/3$



$2\pi - \frac{\pi}{3}$

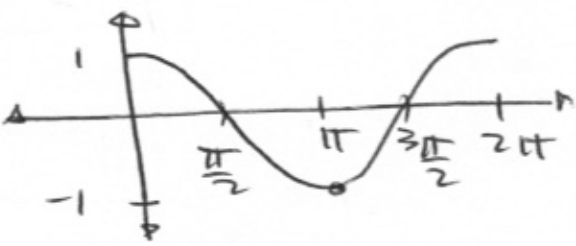
$\frac{7\pi}{3}$

13. $\cos\frac{x}{2}+1=0$;

$\cos\frac{x}{2} = -1$

let $\theta = \frac{x}{2}$

$\cos\theta = -1$



$\theta = \pi$,

$\theta = \pi + 2n\pi$

$\theta = (2n+1)\pi$

but $\theta = \frac{x}{2}$

$\frac{x}{2} = (2n+1)\pi$

$\boxed{x = (2n+1)\cdot 2\pi}$

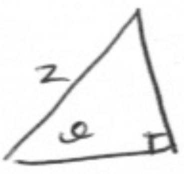
14. $2sin(3x) - \sqrt{3} = 0$ over $[0,2\pi]$

$2\sin(3x) = \sqrt{3}$

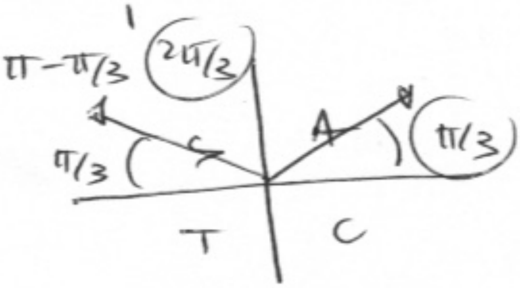
$\sin(3x) = \frac{\sqrt{3}}{2}$

let $\theta = 3x$

$\sin\theta = \frac{\sqrt{3}}{2}$ $\frac{opp}{hyp}$



$\theta = \frac{\pi}{3}$



$\theta_1 = \frac{\pi}{3} + 2n\pi$

$\theta_2 = \frac{2\pi}{3} + 2n\pi$

but $\theta = 3x$:

$3x_1 = \frac{\pi}{3} + 2n\pi$

$\cancel{x_1 = 3\left[\frac{\pi}{3} + 2n\pi\right]}$

$\cancel{x_1 = \pi + 6n\pi}$

$\cancel{\boxed{x_1 = \pi(6n+1)}}$

$\cancel{3x_2 = \frac{2\pi}{3} + 2n\pi}$

$x_1 = \frac{\frac{\pi}{3}}{3} + \frac{2n\pi}{3}$

$x_1 = \frac{\pi}{9} + \frac{2n\pi}{3}$

$\cancel{x_1 = \frac{\pi}{9}\left[1 + 6n\pi\right]}$

$x_1 = \frac{\pi}{9}\left[1+6n\right]$

$\cancel{x_2} =$

$$\frac{\cancel{3}x_2}{\cancel{3}} = \frac{\frac{2\pi}{3} + 2n\pi}{3}$$

$$x_2 = \frac{\frac{2\pi}{3}}{3} + \frac{2n\pi}{3}$$

$$x_2 = \frac{2\pi}{9} + \frac{2n\pi}{3}$$

$$x_2 = \frac{2\pi}{9}\left[1 + \frac{3n}{1}\right]$$

$$\boxed{x_2 = \frac{2\pi}{9}(1+3n)}$$

$$\boxed{x_1 = \frac{\pi}{9}(1+6n)}$$

$n=0$; $x_1 = \frac{\pi}{9}$ $\quad x_2 = \frac{2\pi}{9}$

$n=1$; $x_1 = \frac{8\pi}{9}$ $\quad x_2 = \frac{7\pi}{9}$

$n=2$; $x_1 = \frac{14\pi}{9}$ $\quad x_2 = \frac{13\pi}{9}$

$n=3$; $\cancel{x = \frac{20\pi}{9}}$ $\quad x_2 = \cancel{\frac{19\pi}{9}}$

ie, $\frac{\pi}{9}, \frac{8\pi}{9}, \frac{14\pi}{9}, \frac{2\pi}{9}, \frac{7\pi}{9}, \frac{13\pi}{9}$