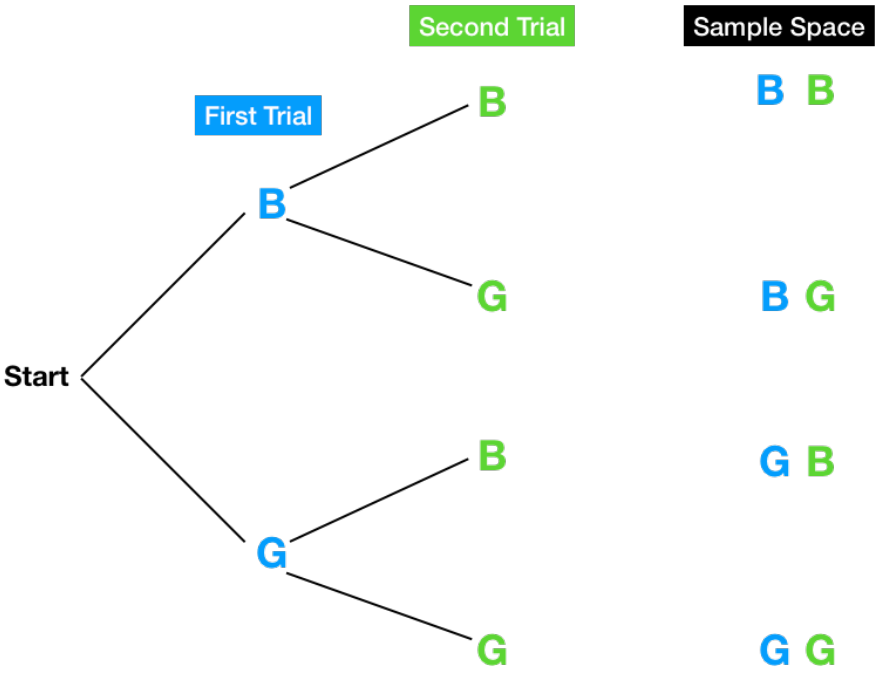
Binomial Probability Distribution

We use this distribution when dealing with a special type of experiment that involves counting success out of a fixed number of trials. Each trial is independent with a probability of success for a single trial.

Consider the experiment of having two children. The tree diagram is as follows.



This process assumes we have one child at a time and each trial is independent of one another. We let $x = \#\ of\ boys$ when having two children be our discrete random variable. We can create the following probability distribution by following our tree diagram.

x	P(x)
0	0.25
1	0.5
2	0.25
Sum	1

What if we want to compute probabilities for couples that want to have **4 children or more**? We can generalize this process by using the Binomial Probability Formula.

$$P(x) = nC_x p^x (1-p)^{n-x}$$

However, we must define each variable in our formula (n,x,p) and describe how this formula works. This formula is based on the concept of independent trials.

n is the number of trials.

x is the number of **successes** out of **n trials**.

p is the probability of a success for a **single trial**.

$q = 1 - p$ is the complement of p.

The key to using the Binomial Probability Formulas is to consider a **single trial**.

Having Four Children
A couple plans on having 4 children, what's the probability of having:

1. No boys?
2. One boy?
3. Two boys?
4. Three boys?
5. Four boys?

In this case, the number of trials is 4, that is $\boldsymbol{n = 4}$

Let x be the number of boys (successes) so that our questions can be posed in terms of the following values of x.

No boys
$x = 0$

One boy
$x = 1$

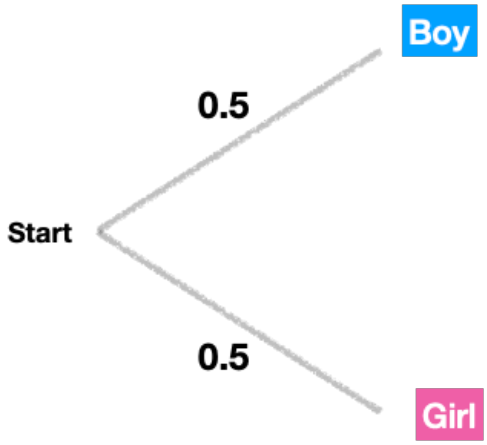
Two boys
$x = 2$

Three boys
$x = 3$

Four boys
$x = 4$

We need to know the value of p for this experiment. In order to determine this value we need to consider a **single trial** for having children.

Single Trial



Since we are counting successes as having boys, the probability of a success for a single trial is $p = \frac{1}{2} = 0.5$

The complement $q = 1 - p$ or $q = 1 - 0.5$ thus $q = 0.5$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

We now have all the information (n, x, p) we need to answer a Binomial Probability Distribution question.

1. **No boys**

$n = 4$
$x = 0$
$p = 0.5$

TI-83 or TI-84
1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(4,0.5,0)

2. **One boy**

$n = 4$
$x = 1$
$p = 0.5$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84 Plus
1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(4,0.5,1)

3. **Two boys**

$n = 4$
$x = 2$
$p = 0.5$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84 Plus
1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(4,0.5,2)

4. **Three boys**

$n = 4$
$x = 3$
$p = 0.5$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84 Plus
1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(4,0.5,3)

5. **Four boys**

$n = 4$
$x = 4$
$p = 0.5$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84 Plus
1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(4,0.5,4)

And of course my favorite questions.

6. At least one boy?

7. More than one boy?

8. Less than two boys?

9. More than two boys?

10. Between one and three boys?

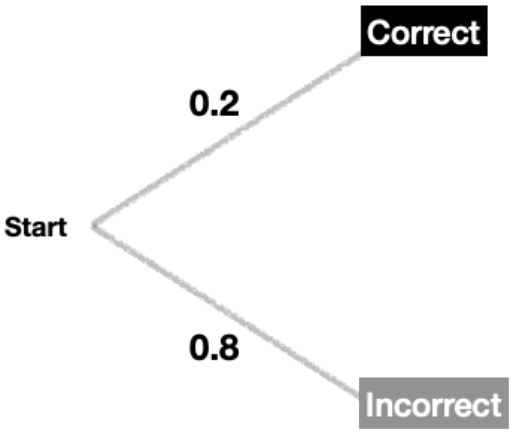
Fact- Just because we have two outcomes, that does not mean that they are equally likely.

$$p \neq q$$

12 Question Multiple Choice Quiz
Every question has 5-possible answers (a-b-c-d-e). If you guess on every question, what's the probability of guess correct on:

11. No questions?
12. One question?
13. Two questions?
14. Three questions?
15. Four questions?

Single Trial



x = number of correct guesses

$n = 12$

$p = \frac{1}{5}$ or $p = 0.2$

$q = 1 - p$ or $q = 0.8$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

11. **No questions**?

$n = 12$
$x = 0$
$p = 0.2$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84
1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(12,0.2,0)

12. **One questions**?

$n = 12$
$x = 1$
$p = 0.2$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84 Plus
1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(12,0.2,1)

13. **Two questions**?

$n = 12$
$x = 2$
$p = 0.2$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84 Plus
1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(12,0.2,2)

14. **Three questions**?

$n = 12$
$x = 3$
$p = 0.2$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84 Plus
1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(12,0.2,3)

15. **Four questions**?

$n = 12$
$x = 4$
$p = 0.2$

$$P(x) = nC_x p^x (1-p)^{n-x}$$

TI-83 or TI-84 Plus
1. Press **2nd** then **vars** to access DISTR (distributions) menu.
2. Select **binompdf** and click **enter**.
3. Enter the values for x, n, and p to complete the command **binompdf(n,p,x)** and press **enter.**

binompdf(12,0.2,4)

And, my favorite questions.

16. At least one correct?

17. At least two correct?

18. More than four correct?

19. Less than ten correct?

20. Between one and four correct?

Four Children

Determine the mean, variance, and standard deviation.

4 Children

x	P(x)
0	0.063
1	0.25
2	0.375
3	0.25
4	0.063

Sum

This Binomial Porbability Distribution also has a way to compute its mean, variance, and standard deviation. In fact, these are the short cut formulas.

Mean $\mu = np$

Variance $\sigma^2 = npq$

Standard Deviation $\sigma = \sqrt{npq}$

Left Handed (select 6 students at random)

Determine the mean, variance, and standard deviation.

6 Students

x	P(x)
0	0.531
1	0.354
2	0.098
3	
4	
5	0.000054
6	0.000001

Sum